Uniform Price and Discriminatory Auctions with Variable Supply

Damian S. Damianov*

University of Texas – Pan American

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Abstract

This paper studies a divisible good auction model in which the seller determines supply after observing bids so as to maximize profit. Bidders have possibly asymmetric downward sloping demand curves and the seller has strictly increasing marginal cost. We show that, in all equilibria of the discriminatory auction, the Walrasian quantities are traded at the Walrasian price. Price discrimination does not materialize in equilibrium because bidders strategically misrepresent their demand, yet this feature eliminates the underpricing equilibria. In the uniform price auction, low-price equilibria exist even when supply is adjustable.

Key Words: variable supply auctions, discriminatory and uniform price auctions, subgame perfect equilibria

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*Department of Economics and Finance, University of Texas – Pan American, 1201 West University Drive, Edinburg, Texas 78541-2999, USA, email: ddamianov@utpa.edu

downward sloping demand curves. Similarly to the aforementioned literature, we assume that bidders are perfectly informed about other bidders' demand and the marginal cost of the seller. The paper provides two new insights on bidding in variable supply auctions.

First, we demonstrate that the discriminatory auction has a unique equilibrium outcome in which the Walrasian quantities are traded at the Walrasian price. Price discrimination does not materialize in equilibrium because bidders strategically misrepresent their true demand. They quote the same (stop-out) price and effectively counterbalance the discriminatory power of the monopolist. Yet, this strategic feature eliminates the underpricing equilibria in the auction. In a model of fixed supply and perfectly elastic demand Back and Zender (1993) demonstrate that bidders in the discriminatory auction will bid a price equal to the value of the good, v, which is assumed common knowledge. We show that more general result holds when supply is adjustable.

Second, we construct an equilibrium example of the uniform price auction in which the stop-out price is below the Walrasian price. Thus, the results relying on the assumption of perfectly elastic demand should be applied with care because they do not extend to a more general formulation of the market game.

Our results establish a ranking of the discriminatory and the uniform price auction in a variable supply model. In practice, both auctions are used. The Treasury departments of Sweden, Switzerland, Finland, Germany and Mexico[5] use a discriminatory auction; the Treasuries of Italy and Norway employ a uniform price auction. In all these countries the Treasuries determine supply ex post. The only variable supply model which discusses both the uniform price and the discriminatory auction is Lengwiler (1999). In this model the seller is privately informed about his constant marginal cost. Because of this assumption in the discriminatory auction bidders virtually do not compete: all bids above the marginal cost are served. This is in contrast to the analysis presented here. Additionally, Lengwiler restricts bidders' choice to the announcement of demand quantities only at two exogenously given price levels (we allow for general left-continuous bid functions), and takes no reference to the Walrasian price. This makes results incomparable, suggests, however, that the assumption of perfectly informed bidders is consequential for the results (ranking in Lengwiler's model is ambiguous).

2 Model and results

A monopolist auctions off a perfectly divisible good to $n \geq 2$ buyers. Each bidder i has continuous and monotonically decreasing demand function $d_i(p)$. The seller has continuous, monotonically increasing and unbounded above marginal cost function $MC(Q)$.

[5] Umlauf (1993) reports that Mexico switched from a uniform price to a discriminatory auction in 1990.

With these preliminaries we characterize equilibrium bidding in the uniform price and the discriminatory auction.

The discriminatory auction

In the discriminatory auction each bidder pays the area under his announced demand curve. Formally, his total payment is $p_S \cdot Q_i + \int_{p_S}^{\infty} b_i(p)\mathrm{d}p$. The next theorem establishes our main result.

Theorem 1. *In all (subgame perfect) equilibria of the discriminatory auction the Walrasian quantities are traded at the Walrasian price. Bidders submit flat demand curves at the Walrasian price, proportionally overstating their actual demand:*

$$b_i^*(p) = \begin{cases} 0 & \textit{for } p > p^w, \\ \lambda \cdot d_i(p^w) & \textit{for } p \leq p^w, \end{cases}$$

where

$$\lambda \geq \frac{D(p^w)}{D(p^w) - \max\limits_{i \in \{1,2,\dots,n\}} \{d_i(p^w)\}}.$$

In equilibrium, bidders overstate their true demand at the Walrasian price by a common factor λ. The role of the threshold value $\frac{D(p^w)}{D(p^w) - \max\limits_{i \in \{1,2,\dots,n\}} \{d_i(p^w)\}}$ is to ensure that no bidder has a deviation that can lower the stop-out price. Indeed, even if one of the bidders lowers his bid, the announced aggregate demand of the other bidders will correspond to the Walrasian quantity or be higher. In such a case the seller will not serve the deviating bidder, and such a deviation will not be profitable. Next, we present a formal proof of the theorem.

Proof. The proof proceeds by contradiction and is organized in four steps. Let us denote the equilibrium stop-out price by p_S^*. In the discriminatory auction the seller acts as a perfectly discriminating monopolist with respect to the received bids, so at the stop-out price he will supply $\min\{B(p_S^*), S(p_S^*)\}$.

Step 1. All bidders announce flat demand curves at the same (stop-out) price p_S^*.

Assume that there exists a bidder i who does not submit (entirely) flat demand at the stop-price. Let the quantity announced at higher prices than the stop-out price be $b_i^{*+}(p_S^*)$. This quantity will be awarded in full, and the bidder will be charged $p_S^* \cdot b_i^{*+}(p_S^*) + \int_{p_S}^{\infty} b_i^*(p)\mathrm{d}p$

Step 4. The strategy profiles, as formulated in the Theorem, are equilibrium profiles. In every equilibrium the Walrasian quantities are traded at the Walrasian price.

It is easily observed that no profitable deviations exist. Since all bidders overstate their demand, if a bidder lowers his bid, he will not be served because the seller can sell the quantity $S(p^w)$ at the price p^w.[8] If a bidder deviates from the quantity at p^w as stated in the Theorem, he will be awarded a quantity different than $d_i(p^w)$, which is unprofitable. Therefore, no other equilibria exist. □

The uniform price auction

In the uniform price auction all bidders pay the stop-out price for each awarded unit. The equilibrium strategy profile stated in Theorem 1 is easily seen to be an equilibrium also in the uniform price auction. The next example illustrates an underpricing equilibrium.

Example 1. *Two bidders participate in a uniform price auction. The demand of each bidder $i \in \{1, 2\}$ is*

$$d_i(p) = \begin{cases} 0 & \text{for } p > 2.2, \\ 1 & \text{for } p \leq 2.2. \end{cases}$$

The marginal cost of the seller is $MC(Q) = Q$. The Walrasian price is $p^w = 2$. The bids

$$b_1^*(p) = \begin{cases} 0 & \text{for } p > 2, \\ 1 & \text{for } p \leq 2, \end{cases}$$

and

$$b_2^*(p) = \begin{cases} 0 & \text{for } p > \sqrt{3}, \\ \sqrt{3} & \text{for } p \leq \sqrt{3}. \end{cases}$$

are equilibrium bids. The seller supplies the quantity $\sqrt{3}$, the stop-out price is $\sqrt{3}$, and the bidders are granted the quantities $Q_1^* = 1$, $Q_2^* = \sqrt{3} - 1$.

Quantities 1 and $\sqrt{3}$ are optimal for the seller. They generate a profit of $\frac{3}{2}$, so we consider the case in which supply is $\sqrt{3}$. Bidder 1 obtains his desired quantity at the stop-out price $\sqrt{3}$. He has no deviation which can lower the stop-out price because, if he submits a bid price below $\sqrt{3}$, he will not be served. In such a case the auctioneer will sell the quantity $\sqrt{3}$ to bidder 2. Bidder 1 obtains his desired quantity, so deviations that lead to different quantities are also not profitable. Observe that in the case of perfectly elastic and unrestricted demand this argument unravels, and low-price equilibria

[8] Observe that if all bidders announce their demand truthfully, then a bidder who slightly lowers his bid price might still be served, and the benefit from the reduction in price might overcompensate the reduction in quantity. So, such a deviation might be profitable. Proportional overstatement of demand eliminates this possibility.

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