BILATERAL BARGAINING: THE CASE OF FIRMS AND WORKERS IN DENMARK*

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Abstract

We employ a simple two-person bargaining model to interpret wage data—demands (offers) by workers (firms) and acceptances by firms (workers). Under two polar-extreme bargaining solutions, we develop a strategy to recover estimates of the marginal-productivity and the opportunity-cost distributions. We then implement this framework using particularly rich data from a sample of Danish firms and workers. Subsequently, we use our estimates to measure the cost of the inefficiencies that arise from the bilateral-monopoly problem under the two alterative bargaining solutions.

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considerable flexibility both in wage setting and regarding employment separations. Specifically, in Denmark, collective wage bargaining was abandoned by most labour unions in the early 1990s, being replaced by individual wage bargaining at the firm level, especially for white-collar workers. This relatively recent development suggests that the bargaining process may be well-approximated by the double-auction model we develop below.

Also, because employees generally do not know their true productivity at any potential firm and employers do not observe their employees' opportunity cost of time, a double-auction game of incomplete information is a natural way to analyze the wage bargaining process under private information. The structural restrictions of this model allow us to estimate otherwise unobserved marginal productivity and opportunity cost distributions. These estimates can be then used to assess the inefficiencies that obtain as a result of privately-held information. By making the distributions depend on observable characteristics, we can also learn about factors that influence the distributions.

Our empirical work is based on a particularly rich data source, the "Ever-Private-Sector" database created by Statistics Denmark and administered by the Center for Corporate Performance at the Aarhus School of Business. This database links employees to their employers and contains detailed wage information as well as a host of variables related to demographic characteristics and labour-market outcomes.

We use this data set to implement the empirical framework that we have developed. Based on our estimates of the distributions of marginal productivity and the opportunity cost of time in alternative uses, we estimate that the inefficiencies that obtain are on the order of ten percent of the wage. Moreover, our estimates indicate that about one-half of observed separations arise because of asymmetric information.

Our empirical framework is closely related to, but different from, the research reported by Elyakime, Laffont, Loisel, and Vuong (1997). In that paper, data from first-price, sealed-bid auctions of timber in the southwest of France were examined. Elyakime *et al.* noted that, in their application, bargaining occurred between the seller and the n potential buyers *after* a first-price, sealed-bid auction. Specifically,

2. A Simple Model of Bilateral Bargaining

We first define a notation and then develop a simple model. This model is very similar to one applied by Hall and Lazear (1984) to the case of labour, as well as a model in a more general setting considered by Chatterjee and Samuelson (1983). An elaborate survey of these models has been undertaken by Ausubel, Cramton, and Deneckere (2002). In our application of the model, a buyer is a firm and a seller is a worker. We assume that the firm knows the value of the marginal product of the worker at that firm; we denote this B. (In what follows, we shall often just refer to B as the marginal productivity of the worker, for short.) We also assume that the worker knows the opportunity cost of his outside alternative; we denote this S. The letters B and S are pnemonic for the values to the *buyer* and the *seller*.[2] In the model, an asymmetry of information exists: each party knows his own valuation, but regards the other party's valuation as an independent draw from an atomless distribution. Denote by $F_B(b)$ the cumulative distribution function of B which has support on $[\underline{b}, \bar{b}]$ where $f_B(b)$ denotes the corresponding probability density function. Similarly, denote by $F_S(s)$ the cumulative distribution function of S which has support on $[\underline{s}, \bar{s}]$ where $f_S(s)$ denotes the corresponding probability density function. Realizations of B and S are denoted by b and s, respectively. We assume that $F_B(\cdot)$ and $F_S(\cdot)$ are common knowledge. In addition, we assume that the firm is risk neutral, while the worker is risk averse, having a von Neumann–Morgenstern utility function U over prospect x which exhibits constant absolute risk aversion (CARA) α, so

$$U(x) = 1 - \exp(-\alpha x) \quad \alpha > 0.$$

We restrict ourselves to two extreme cases. First, the worker sets the wage by announcing a wage demand; the firm then decides whether to employ the worker at that wage. Second, the firm sets the wage by announcing a wage offer; the worker then decides whether to work at that wage. Hereafter, we refer to the first case as Case 1, and the second as Case 2.

[2] Note that, with minor modification, this framework can be used in any situation that has a single seller of an indivisible good who faces a single potential buyer.

To illustrate the mechanics of the solution to this problem, consider an example where B is exponentially distributed, having mean parameter β, so

$$F_B(b) = [1 - \exp(-b/\beta)] \quad b > 0,\ \beta > 0.$$

In this example, the wage-demand function can be solved in closed-form to be

$$W = \omega_1(S) = S + \frac{\log(1 + \alpha\beta)}{\alpha}.$$

The wage-demand function is a constant added to the value of the outside option S. Note, too, that as α gets large, the wage demand gets close to the "full-information" solution where the worker reveals his true value of the outside option.

In Case 2, the firm makes its wage offer w and the worker must then decide whether to accept this offer. The firm at which the worker's marginal productivity is b seeks to maximize expected profit, which is

$$(b - w)F_S(w),$$

the first-order condition for which is

$$w = b - \frac{F_S(w)}{f_S(w)}.$$

When $[F_S(s)/f_S(s)]$ satisfies the Monotone Likelihood-Ratio Property (MLRP), a unique solution to this first-order condition obtains. The worker must decide whether to accept this offer. If the worker rejects the offer, then he receives the utility of the alternative value of his time $U(s)$, while if he accepts the offer, then he receives the utility of the wage $U(w)$. Hence, the worker will accept the wage offer w when it is above the value of his outside option s. Consequently, truth-telling, revealing the opportunity cost of his time, is the optimal strategy of a worker having an outside option with value s.

To illustrate the mechanics of the solution in this case, suppose $\log S$ is distributed according to a Gumbel distribution, having location parameter μ and scale parameter σ, so

$$F_{\log S}(\log s) = \exp\left\{-\exp\left[-(\log s - \mu)/\sigma\right]\right\}.$$

Below, we describe only our econometric analysis of Case 1—when the worker makes a wage demand and the firm either accepts or rejects this demand—because an analysis of Case 2—when the firm makes an offer and the worker either accepts or rejects this offer—is virtually identical. Of course, the actual bargain may obtain somewhere between these two cases. Later, we present empirical evidence regarding Case 2, while in a section of the appendix we provide a complete derivation of the likelihood function in Case 2.

As mentioned, we only observe the wage when both parties agree. In Case 1, the probability of this event is

$$\Pr[B \geq \omega_1(S)] = \Pr[B - \omega_1(S) \geq 0]$$

where

$$W = \omega_1(S).$$

Of course, the probability of a separation is

$$\Pr[B < \omega_1(S)] = \Pr[B - \omega_1(S) < 0].$$

Essentially, in the vocabulary of Amemiya (1985), this is a *Type-5 Tobit* model. To see this, let

$$Y = B - \omega_1(S).$$

We can then rewrite the data-generating process (DGP) as

$$Y = B - \omega_1(S);$$

$$W = \begin{cases} \omega_1(S) & \text{if } Y \geq 0 \\ 0 & \text{otherwise;} \end{cases}$$

$$S = \begin{cases} S & \text{if } Y < 0 \\ 0 & \text{otherwise.} \end{cases}$$

Suppose $F_S(\cdot)$ belongs to a parametric family $F_S(\cdot|\boldsymbol{\varphi})$ indexed by the unknown vector $\boldsymbol{\varphi}$ and $F_B(\cdot)$ belongs to another parametric family $F_B(\cdot|\boldsymbol{\lambda})$ indexed by the unknown vector $\boldsymbol{\lambda}$.

$\boldsymbol{\theta}$ which equals $(\boldsymbol{\varphi}^\top, \boldsymbol{\lambda}^\top)^\top$. Characterize the set of feasible values of $\boldsymbol{\theta}$, that are consistent with the data, by

$$\boldsymbol{\Theta}_T^* = \{\boldsymbol{\theta} \in \boldsymbol{\Theta}^0 | \underline{w}(\boldsymbol{\theta}, \boldsymbol{X}_t) \leq w_t \leq \bar{w}(\boldsymbol{\theta}, \boldsymbol{X}_t), t = 1, \ldots, T\}$$

where $\underline{w}(\boldsymbol{\theta}, \boldsymbol{X}_t)$ and $\bar{w}(\boldsymbol{\theta}, \boldsymbol{X}_t)$ denote the lower and the upper support for the distribution of observed wages, respectively. Here, $\boldsymbol{X}_t$ denotes an observation-specific covariate vector that can affect the lower and upper bounds of support through $f_B(\cdot|\boldsymbol{\lambda})$ and $f_S(\cdot|\boldsymbol{\varphi})$. The solution of the maximization problem is then given by

$$\max_{\boldsymbol{\theta}} \ \log \mathcal{L}(\boldsymbol{\theta}) \quad \text{subject to} \quad \boldsymbol{\theta} \in \boldsymbol{\Theta}_T^*.$$

We do not discuss here the asymptotic distribution of this maximum-likelihood estimator, which is nonstandard. However, details concerning it can be found in Donald and Paarsch (1996), Hong (1998), and Chernozhukov and Hong (2004).

4. An Application to the Danish Labour Market

Within the European context, the Danish labour market is characterized by considerable flexibility both in wage setting and regarding employment separations. This makes the Danish labour market a useful "test-site" to examine various theories concerning labour markets. Lay-offs of workers in Denmark are subject to severance payments, *Fratrædelsesgodtgørelse*, which are one month's salary for job tenure less than or equal to twelve years, and up to three months' salary for job tenure above eighteen years. Firms must give reasons for lay-offs, but these reasons are often vague and the procedure is perfunctory. Collective wage bargaining was abandoned by almost all occupations in the late 1980s, having been replaced in the 1990s by individual bargaining at the firm level, especially for white-collar workers. This recent development suggests that the bargaining process may be well-approximated by the double-auction model outlined above.

In our research, we estimated empirical specifications concerning the two polar extremes. It may be that the true solution lies somewhere in between these two extremes; we consider our research to be a first step in the process of developing more

who is the employee? For the same reason, we eliminated top-level executives who may have some discretion in setting their wages. Also, in an effort to decrease heterogeneity in the sample, we eliminated workers who were not members of any labour union, or for whom union membership was not identified in the data because unemployment benefits are conditional on union membership. Finally, in an effort to avoid mismeasuring wages, we also eliminated workers who took temporary leaves. To wit, those workers who did not separate from the firm, but who did receive unemployment benefits or who were registered as unemployed in 2000.

Despite our care in "cleaning" the data, some unusual observations remained. For example, a worker existed who earned just 10DKK per year, about US$2, even though he was eligible for unemployment benefits on the order of about 143, 500DKK. We believe that these unusual observations arise because of recording and/or reporting errors—measurement error. Below, we introduce measurement error into our econometric specification to deal with this problem.

In Table 4.1, we present the descriptive statistics concerning the final data set that we used.

4.2. Empirical Implementation

We consider Case 1 first. Later, for the purposes of comparison, we examine Case 2. In what follows, we assume that the value of the marginal product of a worker B follows the Weibull law, having scale parameter β_t which varies across workers and shape parameter ρ that does not. We assume that

$$\beta_t = \exp(\boldsymbol{x}_t \boldsymbol{\gamma})$$

where $\boldsymbol{x}_t$ is a vector of covariates that is conformable to the unknown parameter vector $\boldsymbol{\gamma}$. We chose the Weibull distribution for several reasons. First, simplicity—it is fairly easy to interpret the equilibrium wage demand. Second, computational parsimony. Under the Weibull assumption, the cumulative distribution function of B_t, conditional on $\boldsymbol{X}$, is given by

$$F_{B|\boldsymbol{X}}(b_t|\boldsymbol{x}_t) = 1 - \exp\left[-\left(\frac{b_t}{\beta_t}\right)^{\rho}\right] \quad 0 < \beta_t,\ \rho > 0,$$

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where

$$\mu_t = \exp(\boldsymbol{x}_t \boldsymbol{\delta}).$$

Here, $\boldsymbol{x}_t$ is a vector of covariates that is conformable to the unknown parameter vector $\boldsymbol{\delta}$. We also allow α to vary with covariates according to

$$\alpha_t = \exp(\boldsymbol{x}_t \boldsymbol{\psi}).$$

Under these assumptions, the first-order condition for observation t and Case 1 is given by

$$s_t = w_t - \frac{\log\left(1 + \alpha_t \frac{\beta_t^{\rho}}{\rho w_t^{\rho-1}}\right)}{\alpha_t}.$$

The first-order condition in Case 2 reduces to

$$b_t = w_t \left[1 + \frac{\sigma}{\exp\left(\frac{\mu_t - \log w_t}{\sigma}\right)}\right].$$

To introduce measurement error, we assume that the observed wage $\tilde{W}$ is related to the actual wage W and the measurement error ε according to

$$\tilde{W} = W\varepsilon = \omega_1(S)\varepsilon,$$

in the case of an agreement, and

$$\tilde{S} = S\varepsilon,$$

when no agreement is reached. We assume that the measurement error ε is distributed log-normally with mean one and that ε is independent of B and S. By introducing log-normal measurement error, the support of observed wages no longer depends on the unknown parameters of the model, thus allowing us to use standard first-order asymptotic methods to calculate the asymptotic distribution and standard errors.

Table 4.3
Case 2: Parameter Estimates and Standard Errors

	Location Parameter: of Outside Option μ		Location Parameter: of Marginal Product β	
Covariate	Estimate	Std.Err.	Estimate	Std.Err.
Constant	−0.153	0.019	1.716	0.045
Age	−0.010	0.001	0.018	0.001
Female	−0.268	0.005	−0.369	0.012
Number of Children	0.025	0.002	−0.007	0.005
Copenhagen Area	0.131	0.005	−0.004	0.012
Labor Market Experience	0.013	0.001	0.016	0.001
Tenure at Current Firm	−0.015	0.001	0.030	0.001
Education: Vocational	−0.146	0.009	−0.012	0.018
Education: College	0.203	0.009	0.073	0.020
Education: University	0.357	0.009	0.262	0.020
Industry: Manufacturing	0.112	0.009	0.169	0.025
Industry: Trade	0.160	0.008	−0.038	0.018
Industry: Transport	−0.044	0.013	0.370	0.029
Industry: Other	0.105	0.004	0.370	0.010
Logarithm of Variance Parameter for Outside Option			−1.322	0.007
Logarithm of Shape Parameter of Marginal Product			0.135	0.007
Logarithm of Variance Parameter of Measurement Error			−1.010	0.003

comment. First, our parameter estimate for the indicator covariate for females is negative, both on the value of the outside option and on the marginal product. In addition, we estimate a negative relationship on the risk-aversion parameter. Second, to the extent that tenure at the firm measures firm-specific human capital, we find a positive relationship with the marginal product, but mixed evidence concerning the effect on the value of the outside option. Tenure also seems to have a positive relationship with the risk-aversion parameter. Third, living in the Copenhagen area appears to have a positive relationship on the value of the outside option. This is, perhaps, not all that surprising since Copenhagen is by far the largest city in

6. Summary and Conclusions

We have employed a simple two-person bargaining model to interpret wage data—demands (offers) by workers (firms) and acceptances by firms (workers). Under two polar-extreme bargaining solutions, we developed a strategy to recover estimates of the marginal-productivity and the opportunity-cost distributions. We then implemented this framework using particularly-rich data from a sample of Danish firms and workers. Subsequently, we used our estimates to measure the cost of the inefficiencies that arise from the bilateral-monopoly problem under the two alterative bargaining solutions. In both cases, our estimates suggest nontrivial costs arising from the presence of asymmetric information, about ten percent of the wage. Moreover, our estimates indicate that about one-half of observed separations arise because of asymmetric informaton.

so substituting these density functions into the likelihood function above, we obtain

$$\mathcal{L}_2 = \prod_0 f_S(s) F_B[\omega_2^{-1}(s)] \prod_1 f_W(s) F_S(w).$$