Theory of the Gravity Trade Model

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03/24/11

Abstract

A new framework is adopted, inclusive of shipping costs and tariffs, together with assumptions that are rather less restrictive than those typically employed in this context, thus enabling rather more interesting results to be obtained than are available in the literature.

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1 Introduction

The gravity trade model is essentially an empirical model that explains bilateral trade between two regions in terms of their populations and incomes, and barriers between the two, such as distance apart or tariffs. A typical such model (Frankel, Stein and Wei 1995) is:

$$\log T = a + b\log(Y_A Y_B) + c\log\left(\frac{Y_A}{N_A}\frac{Y_B}{N_B}\right) + d\log S, \tag{1}$$

where T is trade between A and B, Y_A is A's GDP, Y_B is B's GDP, N_A is A's population, N_B is B's population and S is distance between A and B. Henceforth, we call A and B 'countries'.

Although the gravity trade model has been a splendid workhorse for empirical trade models, especially models of aggregate trade, its theoretical foundations are relatively unexplored. After early work by Tinbergen (1962), there have been important contributions by Anderson (1979), Helpman and Krugman (1985), Bergstrand (1989), Deardorff (1998), Everett and Keller (2002), and Anderson and van Wincoop (2003, 2004).

Anderson's (1979) seminal work assumes a uniform Cobb-Douglas utility function, and Anderson and van Wincoop (2003, 2004) a uniform CES utility function. The typical result with a homothetic utility function is that bilateral trade is related to the product of the importer's and exporter's incomes divided by world income. A further strand of the literature (see the other papers mentioned in the previous paragraph), concerns structure. Everett and Keller (2002), for example, finds interesting equivalences among alternative structures. (They do, though, neglect transportation costs, which makes it difficult to interpret their models as true gravity models.)

In this paper, we adopt a fresh approach. We introduce a non-traded good, assume uniform Cobb-Douglas demand functions for tradables and use a condition required for integrability in obtaining comparative statics results. Cobb-Douglas demand functions enable us to relax homothetic preference. We follow others in assuming specialisation in trade. However, we innovate in that ‘locations’ rather than countries are the fundamental unit, so that a single country exports many tradables.

The approach enables us to provide a general treatment of transport costs and leads to an elegant result. A feature is, unlike in Anderson and van Wincoop (2003, 2004), transport costs from the exporter to third countries are not relevant to bilateral trade. The reason is, flexible rather than fixed production is assumed, where however flexible production seems the appropriate assumption to make for use in empirical cross-section studies. A further innovation of the paper is in dealing separately with tariffs, again with complete generality, in a unified multi-regional framework.

2 The Model

Assume there are N ‘locations’ and $N+2$ goods. Goods 1 to N are traded consumption goods, good $N+1$ is a non-traded consumption good, and good $N+2$ is ‘shipping’. Each location has a population of size 1. Location i produces, competitively, goods i, $N+1$ and $N+2$. No other locations produce good i, i.e. traded goods have some local characteristics. Separate local markets exist for the non-traded consumption good, but there is a single market for each traded good, and for shipping. All markets clear.

Let location i produce positive amounts of good k for $k = i$, $N+1$ and $N+2$. All marginal rates of transformation are unity. Thus all goods have the same price, which we set equal to unity.

Traded goods differ in price between locations, both through the cost of shipping and because of tariffs. Let θ_{kj} be the cost of shipping a unit of traded good k from location k to location j, let τ_{kj} be the tariff applied to good k at location j, and let p_{kj} be good k’s price at location j. Then, for $k, j = 1, ..., N$:

$$p_{kj} = (1+\theta_{kj})(1+\tau_{kj}). \tag{2}$$

According to (2), the price of traded good k at location j is the price (i.e. unity) at location k plus the cost of shipping a unit from k to j, scaled up by the tariff applied at j.

Assume location j’s demand for consumption good k is:

$$x_{kj} = F_{kj}(p_j, y_j), \tag{3}$$

where $p_j = (p_{1j}, ..., p_{Nj}, 1)$, and y_j is location j’s income.

We suppose the N locations are partitioned into ‘countries’ and focus on any two of these, A and $A*$, in order to investigate bilateral trade. Let A include N_A locations and $A*$ include N_{A*} locations.

Suppose countries exclusively provide shipping for their own exports. The value of total exports of goods from A to $A*$ (or imports of $A*$ from A), including shipping costs, is:

$$X = \sum_{i \in A} \sum_{j \in A*} (1+\theta_{ij}) F_{ij}(p_j, y_j). \tag{4}$$

2.1 *Cobb-Douglas Demand*

Referring to (3), we suppose that the demand functions for traded goods take the specific form:

$$x_{kj} = dp_{1j}^{\beta} ... p_{k-1,j}^{\beta} p_{kj}^{\beta-1} p_{k+1,j}^{\beta} ... p_{Nj}^{\beta} p_{N+1,j}^{\alpha} y_j^{\delta}, \tag{5}$$

where $\beta < 1$ and $\delta > 0$. I.e. the own-price effect is negative, cross-price effects among traded goods are equal but indeterminate in sign, and traded goods are normal.

In (5), note that for traded goods:

(a) The symmetry condition requires the own-price index to be one less than corresponding cross-price indices.

(b) The homogeneity condition requires:

$$\alpha = 1 - \delta - N\beta. \tag{6}$$

It can be shown that the integrability condition is satisfied when β is small or negative and there is a sufficiently dominant non-traded consumption good; i.e. there then exists a utility function from which these demand functions can be derived. A necessary condition is:

$$N\beta < 1 \tag{7}$$

(see Appendix). An important feature of Cobb-Douglas demand is it allows traded goods to be luxury (or necessary) goods.

Substituting (2) into (5), noting $p_{N+1,j} = 1$, we obtain:

$$\begin{aligned} x_{kj} &= d(1+\theta_{1j})^{\beta}(1+\tau_{1j})^{\beta}...(1+\theta_{k-1,j})^{\beta}(1+\tau_{k-1,j})^{\beta} \quad (8) \\ &\quad (1+\theta_{kj})^{\beta-1}(1+\tau_{kj})^{\beta-1}(1+\theta_{k+1,j})^{\beta}(1+\tau_{k+1,j})^{\beta} \quad (9) \\ &\quad ...(1+\theta_{Nj})^{\beta}(1+\tau_{Nj})^{\beta}y_j^{\delta}. \quad (10) \end{aligned}$$

Substituting (8) into (4), we obtain:

$$\begin{aligned} X &= \sum_{i\in A}\sum_{j\in A*}(1+\theta_{ij})d(1+\theta_{1j})^{\beta}(1+\tau_{1j})^{\beta}... \quad (11) \\ &\quad (1+\theta_{ij})^{\beta-1}(1+\tau_{ij})^{\beta-1}...(1+\theta_{Nj})^{\beta}(1+\tau_{Nj})^{\beta}y_j^{\delta}. \quad (12) \end{aligned}$$

In order to express exports in terms of aggregates, we assume:

(1) Locations in $A*$ have uniform income, i.e. for all $j \in A*$, $y_j = y_{A*}$.

(2) $A*$ applies a uniform tariff to imports from a subset S of locations, where $S \frown A* = O$, either $A \subset S$ or $A \frown S = O$ and S includes N_S locations. I.e. for all $i \in S$ and $j \in A*$, $\tau_{ij} = \tau_{A*}$, while $\tau_{ij} = 0$ otherwise.

Let $\omega = 0$ if $A \subset S$ and $\omega = 1$ if $A \frown S = O$. (9) is replaced by:

$$X = (1+\tau_{A*})^{N_S\beta-\omega}\sum_{i\in A}dy_{A*}^{\delta}\sum_{j\in A*}(1+\theta_{1j})^{\beta}...(1+\theta_{Nj})^{\beta}. \tag{13}$$

Define the average trade cost, θ_j, for shipping goods to location j by:

$$(1+\theta_j)^{N\beta} = (1+\theta_{1j})^{\beta}...(1+\theta_{Nj})^{\beta}. \tag{14}$$

Then, substituting (11) into (10):

$$X = (1+\tau_{A*})^{N_S\beta-\omega}\sum_{i\in A}dy_{A*}^{\delta}\sum_{j\in A*}(1+\theta_j)^{\sum_{k=1}^{N}\beta}. \tag{15}$$

Define the average trade cost, θ, for shipping goods to $A*$ by:

$$1+\theta = \sum_{k=1}^{N}(1+\theta_j). \tag{16}$$

Substituting (13) into (12):

$$X = (1+\tau_{A*})^{N_S\beta-\omega}(1+\theta)^{N\beta} d N_A N_{A*} y_{A*}^{\delta}. \tag{17}$$

Let

$$\begin{aligned}
\gamma &= (1+\tau_{A*})^{N_S\beta-\omega}(1+\theta)^{N\beta} d && (18)\\
Y_{A*} &= \sum_{j\in A*} y_j && (19)\\
&= N_{A*}y_{A*}. && (20)
\end{aligned}$$

Substituting (15) and (16) into (14) gives:

$$X = \gamma N_A N_{A*}^{1-\delta} Y_{A*}^{\delta}. \tag{21}$$

3 Conclusion

A striking feature of many theoretical or empirical gravity equations is the symmetric treatment of the two focus countries (for example, see equation (1)). We find no justification for this. For both countries, importer and exporter, measures of 'size' matter, but for different reasons. A point to note, however, is that, although not appearing in (17), Y_A replaces N_A in the gravity equation on relaxing the assumption that all locations produce shipping. We then have:

$$X = \gamma Y_A N_{A*}^{1-\delta} Y_{A*}^{\delta}. \tag{22}$$

where γ depends on prices as well as tariffs and trade costs.

The unit coefficient on A's population (the exporter) reflects the way the model relates variety of goods to population. Again this would change, due to a 'crowding' effect, if the assumption that all locations produce shipping were relaxed. (17) shows that if tradables are luxury goods then, in log-linear form, the coefficient on $A*$'s income (the importer) is greater than unity, and the coefficient on $A*$'s population is negative.

The model distinguishes the costs of shipping from different locations, including those within $A*$, to locations in $A*$. A feature is the symmetric effect that all shipping costs to locations in $A*$ have, given that shipping costs are included in A's exports. Tariffs have the effect of raising A's exports if A is within $A*$'s tariff wall and tradables are substitutes ($\beta > 0$) but not if they are complements ($\beta < 0$) , and lowering A's exports if A is outside $A*$'s tariff wall.

Other barriers to trade, such as cultural differences or the absence of a common frontier, can be dealt with within the above framework, but deserve detailed treatment.

4 Appendix

Lemma

Define A of order n by

$$A = \begin{bmatrix} a & b & b & .. & b \\ b & a & b & .. & b \\ b & b & a & .. & b \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b & b & b & .. & a \end{bmatrix}.$$

A is negative semi-definite if

$$b \geq a$$

and

$$b \leq -\frac{1}{n-1}a.$$

Subtract the second row from the first, the third from the second, etc., to obtain

$$B = \begin{bmatrix} a-b & b-a & 0 & .. & 0 \\ 0 & a-b & b-a & .. & 0 \\ 0 & 0 & a-b & .. & 0 \\ \vdots & \vdots & \vdots & .. & \vdots \\ b & b & b & .. & a \end{bmatrix}.$$

Then add the first column of B to the second, the second to the third, etc., to obtain

$$C = \begin{bmatrix} a-b & 0 & 0 & .. & 0 \\ 0 & a-b & 0 & .. & 0 \\ 0 & 0 & a-b & .. & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b & 2b & 3b & .. & a+(n-1)b \end{bmatrix}$$

$$\begin{aligned} & \det(A) \\ = \ & \det(C) \\ = \ & (a-b)^{n-1}[a+(n-1)b] \end{aligned}$$

In the model, let

$$e = dp_1^{\beta}p_2^{\beta}...p_N^{\beta}p_{N+1}^{\alpha}y^{\delta}.$$

Note that e is expenditure on a traded good. The substitution matrix for traded goods is:

$$
\begin{aligned}
&\begin{bmatrix}
\frac{\partial x_1}{\partial p_1}+x_1\frac{\partial x_1}{\partial y} & \frac{\partial x_1}{\partial p_2}+x_2\frac{\partial x_1}{\partial y} & .. & \frac{\partial x_1}{\partial p_N}+x_N\frac{\partial x_1}{\partial y} \\
\frac{\partial x_2}{\partial p_1}+x_1\frac{\partial x_2}{\partial y} & \frac{\partial x_2}{\partial p_2}+x_2\frac{\partial x_2}{\partial y} & .. & \frac{\partial x_2}{\partial p_N}+x_N\frac{\partial x_2}{\partial y} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{\partial x_N}{\partial p_1}+x_1\frac{\partial x_N}{\partial y} & \frac{\partial x_N}{\partial p_2}+x_2\frac{\partial x_N}{\partial y} & .. & \frac{\partial x_N}{\partial p_N}+x_N\frac{\partial x_N}{\partial y}
\end{bmatrix} \\
= &\begin{bmatrix}
(\beta-1)\frac{e}{p_1^2}+\delta\frac{e^2}{p_1^2 y} & \beta\frac{e}{p_1p_2}+\delta\frac{e^2}{p_1p_2y} & .. & \beta\frac{e}{p_1p_N}+\delta\frac{e^2}{p_1p_Ny} \\
\beta\frac{e}{p_2p_1}+\delta\frac{e^2}{p_2p_1y} & (\beta-1)\frac{e}{p_2^2}+\delta\frac{e^2}{p_2^2 y} & .. & \beta\frac{e}{p_2p_N}+\delta\frac{e^2}{p_2p_Ny} \\
\vdots & \vdots & \vdots & \vdots \\
\beta\frac{e}{p_Np_1}+\delta\frac{e^2}{p_Np_1y} & \beta\frac{e}{p_Np_2}+\delta\frac{e^2}{p_Np_2y} & .. & (\beta-1)\frac{e}{p_N^2}+\delta\frac{e^2}{p_N^2 y}
\end{bmatrix} \\
= &\begin{bmatrix}
\frac{1}{p_1} & 0 & .. & 0 \\
0 & \frac{1}{p_2} & .. & 0 \\
\vdots & \vdots & \vdots & \vdots \\
0 & 0 & .. & \frac{1}{p_N}
\end{bmatrix}
\begin{bmatrix}
(\beta-1)e+\delta\frac{e^2}{y} & \beta e+\delta\frac{e^2}{y} & .. & \beta e+\delta\frac{e^2}{y} \\
\beta e+\delta\frac{e^2}{y} & (\beta-1)e+\delta\frac{e^2}{y} & .. & \beta e+\delta\frac{e^2}{y} \\
\beta e+\delta\frac{e^2}{y} & \beta e+\delta\frac{e^2}{y} & .. & \beta e+\delta\frac{e^2}{y} \\
\beta e+\delta\frac{e^2}{y} & \beta e+\delta\frac{e^2}{y} & .. & (\beta-1)e+\delta\frac{e^2}{y}
\end{bmatrix} \\
&\begin{bmatrix}
\frac{1}{p_1} & 0 & .. & 0 \\
0 & \frac{1}{p_2} & .. & 0 \\
\vdots & \vdots & \vdots & \vdots \\
0 & 0 & .. & \frac{1}{p_N}
\end{bmatrix}.
\end{aligned}
$$

By the lemma, the substitution matrix is negative semi-definite if:

$$
\begin{aligned}
\beta+\delta\frac{e}{y} &\leq -\frac{\beta-1+\delta\frac{e}{y}}{N-1} && (23) \\
\beta+\delta\frac{e}{y} &\leq \frac{1}{N}. && (24)
\end{aligned}
$$

Thus, for β small or negative, and a sufficiently dominant non-traded good, for example $N = 10$, $\beta = 0.05$, $\delta = 1$ and $e \leq 0.05y$, the system of demand functions for traded goods is integrable. A necessary condition is:

$$N\beta < 1.$$

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