

A New Test for Seasonal Dummies

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Abstract

In this paper, we establish the limit theory of the seasonal KPSS test [The seasonal KPSS statistic, *Econom. Bull.* 3 (2006), pp. 1-9] under the null by involving seasonal dummies. Taking these variables into account can be advantageous on different levels. The seasonal KPSS test can be interpreted as a test of deterministic seasonality and it could be used complementarily with seasonal unit root tests to rigorously analyse the dynamic properties of time series. Moreover, the inclusion of seasonal dummies provides the test with an explicit model-based regression that in itself constitutes a compulsory support for its limit theory.

Keywords: KPSS test, deterministic seasonality, Brownian motion

JEL classification: C32 Time series models

1 Introduction

The use of seasonally unadjusted data is increasing at least in empirical studies. This is due to a number of reasons. In fact, it is currently argued that seasonal adjustment distorts inference in dynamic models (e.g., seasonal unit roots can be seriously affected if one has to work with seasonally adjusted data). In this respect, Ghysels and Perron (1993) showed that seasonal adjustment filters affect finite sample distributions of the unit root test statistics under the null hypothesis. Furthermore, because seasonal component is an unobserved part of time series, it

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must be taken into consideration as its elimination can result detrimental information loss. It has also been realized in several cases that the seasonal and the other systematic components, such as trend and cycle, are inseparable. From an economic point of view, this could be attributed to the fact that economic propagation mechanism transmitting seasonal fluctuations from exogenous to endogenous variables is systematically related to that transmitting business cycle fluctuation. This is what Beaulieu, MacKie Mason and Miron (1992) as well as Miron (1996) have shown in international economic aggregates such as output, labor input, interest rates, and prices. Canova and Ghysels (1994), in an empirical study of U.S. macroeconomic time series, similarly found that seasonality tends to differ across the business cycle stages of recessions and expansions. Consequently, a forced seasonal adjustment can lead to inaccurate previsions. As a matter of course, inappropriate decisions will be chosen.

Various reasonable models of seasonality are conceivable. As highlighted by Canova and Hansen (1995), the first approach is modelling seasonality as deterministic. This approach is generally adopted by macroeconomists, as evident in the example of Barsky and Miron (1989). The second approach is considering seasonality as deterministic process along with its stationary stochastic pattern, as shown by Canova (1992). The third approach is to consider seasonality as stochastic by allowing for seasonal unit roots. A famous testing framework proposed by Hylleberg *et al.* [HEGY] (1990) distinguishes between unit roots at different seasonal frequencies. They took the null hypothesis of seasonal nonstationarity induced by the presence of seasonal unit root(s). The subsequent rejection of their null hypothesis implies a strong result that the series exhibit a stationary seasonal pattern. However, their test is somewhat flawed due to its low power in moderate samples sizes. In this case, the other complementary type of tests with null hypothesis of stationary seasonality can provide convenient decisions. Several authors have contributed to the establishment of tests of this type: we can refer to Canova and Hansen (1995) as well as Caner (1998). In this respect, the rejection of the null hypothesis would imply the strong evidence that the data are indeed nonstationary. Another reason that supports the use of such tests is the necessity of testing the dummy variable model bearing in mind the cost of spurious deterministic seasonality, as shown by Franses, Hylleberg and Lee (1995).

In this paper, we include seasonal dummies in the definition of the seasonal KPSS statistic proposed by Lyhagen (2006). Such an inclusion can be advantageous in several ways. It provides a model-based regression to the test, which is contrary to Lyhagen's analysis that didn't establish its limit theory through an explicit model. Additionally, our establishment of

the asymptotic theory of the test in presence of seasonal dummies proposes a new testing framework for deterministic seasonality.

The outline of this paper is as follows. In second section, some preliminaries of the seasonal KPSS test are given. The asymptotic results of this test in presence of seasonal dummies are detailed in the third section. A Monte Carlo simulation investigating small sample properties of the size and power is exposed in the next section. We conclude our findings in the last section.

2 Seasonal KPSS Test: Preliminaries

Consider a quarterly data y_t . We choose this frequency observation because it affords a clear analysis. Note, however, that our test can be extended to other seasonal series (e.g. monthly or daily data) by simply defining seasonal unit roots according to their corresponding seasonal frequencies. If our goal is to test presence of the unit root of -1, it would be suitable to filter the series with an adequate filter to isolate the effects of the other unit roots. In other words, the test will be applied to the transformed series:

$$y_t^{(1)} = (1 - L + L^2 - L^3)y_t, \text{ where } L \text{ is the lag operator.}$$

Next, we test a unit root of -1 in the series below

$$y_t^{(1)} = x_t' \beta + r_t + u_t, \quad t = 1, \dots, T, \quad (1)$$

with $T = 4N$, $\beta' x_t = \sum_{i=1}^4 a_i D_{it}$ and the shorthand notation $D_{it} = \delta(i, t - 4[(t-1)/4])$, where we use $[\cdot]$ for the largest integer function and $\delta(i, j)$ for Kronecker's δ function. The term u_t is zero mean weakly dependent process with autocovariogram $\gamma_h = E(u_t u_{t+h})$ and strictly positive long run variance ω_u^2 .

The component r_t is drawn by the following process

$$r_t = -r_{t-1} + v_t, \quad (2)$$

where v_t is zero mean weakly process with variance σ_v^2 and long run variance $\omega_v^2 > 0$.

The transformation needed to carry out the seasonal KPSS test for complex unit roots $\pm i$ is indicated by the following variable: $y_t^{(2)} = (1 - L^2)y_t$. The test of such complex unit roots is based on the following regression

$$y_t^{(2)} = x_t' \lambda + c_t + e_t, \quad (3)$$

where e_t is zero mean weakly dependent process with long run variance $\omega_e^2 > 0$ and $\lambda' x_t = \sum_{i=1}^4 b_i D_{it}$. The component c_t is drawn by

$$c_t = -c_{t-2} + \varepsilon_t, \quad (4)$$

where ε_t is another zero mean weakly dependent process with variance σ_ε^2 and strictly positive long run variance ω_ε^2 .

The addition of the deterministic components in (1) and (3) is important since it enables us to classify the seasonal KPSS test as a test of deterministic seasonality. However this test proceeds in two stages: First, we test the unit root -1, then we test the complex roots where the null hypothesis will be specified thereafter. If the presence of these roots is rejected, we can affirm that the seasonality is deterministic.

The seasonal KPSS test like that standard is also an LM test. Hence, the null hypothesis corresponding to the test of the unit root of -1 is $H_0: \sigma_v^2 = 0$ under which

$$y_t^{(1)} = x_t' \beta + u_t, \quad (5)$$

is trend stationary after seasonal mean correction. Under the alternative $H_1: \sigma_v^2 > 0$, $y_t^{(1)}$ has a unit root corresponding to Nyquist frequency. Let \tilde{u}_t , $t = 1, 2, \dots, T$ be the residuals obtained from least squares applied to Eq. (5). We can nonparameterically construct a consistent estimator of ω_u^2 using these residuals and the Bartlett kernel method as follows

$$\tilde{\omega}_u^2(l) = T^{-1} \sum_{t=1}^T \tilde{u}_t^2 + 2T^{-1} \sum_{k=1}^l w(k, l) \sum_{t=k+1}^T \tilde{u}_t \tilde{u}_{t-k}, \quad (6)$$

with weight function $w(k, l) = 1 - \frac{k}{l+1}$ and l is a lag truncation parameter such $l \rightarrow \infty$ as

$T \rightarrow \infty$ and $l = o(n^{1/2})$. Andrews (1991) showed that such a truncation lag produces good results

in practice and which was used by Kwiatkowski et al. (1992). In the same way, the null assumption corresponding to the test of the complex roots is $H_0 : \sigma_\varepsilon^2 = 0$, under which

$$y_t^{(2)} = x_t' \lambda + e_t. \quad (7)$$

Using the residuals \tilde{e}_t , obtained from the least squares applied to Eq. (7), we can construct the Bartlett kernel estimator of ω_e^2 as follows

$$\tilde{\omega}_e^2(l) = T^{-1} \sum_{t=1}^T \tilde{e}_t^2 + 2T^{-1} \sum_{k=1}^l w(k,l) \sum_{t=k+1}^T \tilde{e}_t \tilde{e}_{t-k}. \quad (8)$$

Define the partial sums $\tilde{S}_t = \sum_{j=1}^t e^{i\pi j} \tilde{u}_j$ and $\tilde{P}_t = \sum_{j=1}^t e^{i\theta j} \tilde{e}_j$, with $\theta = \pi/2$ or $3\pi/2$.

Thus, the statistic of the test for unit root of -1 is

$$\eta^{(-1)} = \frac{1}{T^2} \frac{\sum_{t=1}^T \tilde{S}_t \overline{\tilde{S}_t}}{\tilde{\omega}_u^2(l)}. \quad (9)$$

This statistic is written, for the complex unit roots, as

$$\eta^{(\pm i)} = \frac{1}{T^2} \frac{\sum_{t=1}^T \tilde{P}_t \overline{\tilde{P}_t}}{\tilde{\omega}_e^2(l)}, \quad (10)$$

where $\overline{\tilde{S}_t}$ and $\overline{\tilde{P}_t}$ are the conjugate numbers of respectively \tilde{S}_t and \tilde{P}_t .

3 Asymptotic results

The next theorem gives the asymptotic distribution of $\eta^{(1)}$ and $\eta^{(\pm i)}$.

Theorem

- a) Under $H_0 : \sigma_v^2 = 0$, $\eta^{(-1)} \rightarrow_d \int_0^1 V(r)^2 dr$, where $V(r)$ is a standard Brownian bridge, “ \rightarrow_d ” denotes weak convergence in probability and $r \in [0,1]$.

b) Under $H_0 : \sigma_\varepsilon^2 = 0, \eta^{(\pm i)} \rightarrow_d \frac{1}{2} \int_0^1 [V_R^c(\tau)^2 + V_I^c(\tau)^2] d\tau$, where $V_R^c(\tau)$ and $V_I^c(\tau)$ are two independent standard Brownian bridges and $\tau \in [0,1]$.

Proof.

We start by proving part a) of the theorem, in accordance with the methodology of Jin and Phillips (2002) who found that seasonal dummies leave the asymptotic theory of the KPSS test intact. Given the mirror image of negative unit roots, we have

$\frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j u_j \rightarrow_d B(r)$ where $B(r)$ is a Brownian motion. We can write the standardized

partial sum process as follows

$$\begin{aligned} \tilde{S}_{[Tr]}^* &= \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} e^{i(\pi j)} \tilde{u}_j = \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j \tilde{u}_j = \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j [u_j - x_j'(\tilde{\beta} - \beta)] \\ &= \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j u_j - \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j x_j' (X'X)^{-1} (X'u), \end{aligned}$$

where $r \in [0,1]$. Hence, we can obtain

$$\tilde{S}_{[Tr]}^* = \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j u_j - \left(\frac{\sum_{j=1}^{[Tr]} (-1)^j x_j'}{T} \right) \left(\frac{X'X}{T} \right)^{-1} \left(\frac{X'u}{\sqrt{T}} \right). \quad (11)$$

We also have $\frac{1}{T} \sum_{j=1}^{[Tr]} (-1)^j x_j' \rightarrow \left(-\frac{r}{4}, \frac{r}{4}, -\frac{r}{4}, \frac{r}{4} \right)$, $T^{-1} X'X \rightarrow (1/4)I_4$, and

$$\frac{1}{\sqrt{T}} X'u = \frac{1}{2} \frac{1}{\sqrt{T}} \begin{pmatrix} \sum_{j=1}^{\frac{T}{4}} u_{4j-3} \\ \sum_{j=1}^{\frac{T}{4}} u_{4j-2} \\ \sum_{j=1}^{\frac{T}{4}} u_{4j-1} \\ \sum_{j=1}^{\frac{T}{4}} u_{4j} \end{pmatrix} \rightarrow \frac{1}{2} \begin{pmatrix} B_{u.1} \\ B_{u.2} \\ B_{u.3} \\ B_{u.4} \end{pmatrix} \equiv \frac{1}{2} N \left(0, \begin{pmatrix} \omega_0 & \omega_1 & \omega_2 & \omega_3 \\ \omega_1 & \omega_0 & \omega_1 & \omega_2 \\ \omega_2 & \omega_1 & \omega_0 & \omega_1 \\ \omega_3 & \omega_2 & \omega_1 & \omega_0 \end{pmatrix} \right), \quad (12)$$

where $B_{u.i}(1) =_d N(0, \omega_0)$, $i = 1, \dots, 4$, and where precisely

$$\omega_0 = \gamma_0 + 2 \sum_{h=1}^{\infty} \gamma_{4h}, \omega_1 = \sum_{h=1}^{\infty} \gamma_{2h-1} = \omega_3, \text{ and } \omega_2 = 2 \sum_{h=1}^{\infty} \gamma_{4h-2}.$$

It follows that

$$\begin{aligned} \left(\frac{\sum_{j=1}^{[T\tau]} (-1)^j x_j'}{T} \right) \left(\frac{X'X}{T} \right)^{-1} \left(\frac{X'u}{\sqrt{T}} \right) &\rightarrow_d \left(-\frac{r}{4}, \frac{r}{4}, -\frac{r}{4}, \frac{r}{4} \right) \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} \frac{1}{2} \begin{pmatrix} B_{u.1}(1) \\ B_{u.2}(1) \\ B_{u.3}(1) \\ B_{u.4}(1) \end{pmatrix} \\ &\rightarrow_d \frac{1}{2} r [-B_{u.1}(1) + B_{u.2}(1) - B_{u.3}(1) + B_{u.4}(1)] \\ &\rightarrow_d \frac{1}{2} r [-B_1(1) + B_2(1) - B_3(1) + B_4(1)] = rB(1). \end{aligned}$$

Consequently, we have $\frac{1}{\sqrt{T}} \tilde{S}_{[T\tau]} \rightarrow_d B(r) - rB(1)$, giving

$$T^{-2} \sum_{i=1}^T \tilde{S}_i^2 \rightarrow_d \omega_u^2 \int_0^1 V(r)^2 dr, \quad (13)$$

where $V(r)$ is a standard Brownian bridge process. Since $\tilde{\omega}_u^2(l)$ is a consistent estimate of ω_u^2 , we can easily deduce that $\eta^{(-1)} \rightarrow_d \int_0^1 V(r)^2 dr$.

We will now demonstrate part b) of the theorem. Since complex-valued roots come in conjugate pairs, we only consider the complex root i associated to the frequency $\frac{\pi}{2}$. In this

case, the standardized partial sum process can be written as follows

$$\frac{\tilde{P}_{[T\tau]}}{\sqrt{T}} = \frac{1}{\sqrt{T}} \sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} \tilde{e}_j = \frac{1}{\sqrt{T}} \sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} \tilde{e}_j = \frac{1}{\sqrt{T}} \sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} [e_j - x_j'(\tilde{\lambda} - \lambda)]$$

$$\frac{\tilde{S}_{[T\tau]}}{\sqrt{T}} = \frac{1}{\sqrt{T}} \sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} e_j - \left(\frac{\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} x_j'}{T} \right) \left(\frac{X'X}{T} \right)^{-1} \left(\frac{X'e}{\sqrt{T}} \right).$$

It has been shown from Chan and Wei (1988) that

$$\frac{1}{\sqrt{T}} \sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} e_j \rightarrow_d B^*(\tau),$$

where $B^*(\tau) = B_R^*(\tau) + iB_I^*(\tau)$ and $B_R^*(\tau)$ and $B_I^*(\tau)$ are two independent real Brownian motions. We have also $\frac{1}{T} \sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} x_j' \rightarrow \left(0, -\frac{r}{4}, 0, \frac{r}{4} \right) + i \left(\frac{r}{4}, 0, -\frac{r}{4}, 0 \right)$. Consequently, we can obtain

$$\left(\frac{\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} x_j}{T} \right) \left(\frac{X'X}{T} \right)^{-1} \left(\frac{X'e}{\sqrt{T}} \right) \rightarrow_d 4 \times \left(0, -\frac{r}{4}, 0, \frac{r}{4} \right) I_4 \times \frac{1}{2} \begin{pmatrix} B_{e.1} \\ B_{e.2} \\ B_{e.3} \\ B_{e.4} \end{pmatrix} + i \left[4 \times \left(\frac{r}{4}, 0, -\frac{r}{4}, 0 \right) I_4 \times \frac{1}{2} \begin{pmatrix} B_{e.1} \\ B_{e.2} \\ B_{e.3} \\ B_{e.4} \end{pmatrix} \right],$$

where $B_{e.i}$ are defined like $B_{u.i}$ in Eq.12, $i = 1, \dots, 4$. It follows that

$$\begin{aligned} \left(\frac{\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} x_j}{T} \right) \left(\frac{X'X}{T} \right)^{-1} \left(\frac{X'e}{\sqrt{T}} \right) &\rightarrow_d \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{2} \tau (-B^{(2)}(1) + B^{(4)}(1)) + i \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{2} \tau (B^{(1)}(1) - B^{(3)}(1)) \\ &= \frac{1}{\sqrt{2}} \tau (B_R^*(1) + i B_I^*(1)) = \frac{1}{\sqrt{2}} \tau B^*(1), \end{aligned}$$

where $B^{(i)}(\tau)$, $i = 1, \dots, 4$, $B_R^*(\tau)$ and $B_I^*(\tau)$ are real Brownian motions with both last ones are independent. We then obtain

$$\frac{\tilde{P}_{[T\tau]}}{\sqrt{T}} \rightarrow_d \sqrt{\frac{\omega_e^2}{2}} V^c(\tau), \text{ where } V^c(\tau) \text{ is a complex Brownian bridge which can be written as}$$

follows $V^c(\tau) = V_R^c(\tau) + iV_I^c(\tau)$ with $V_R^c(\tau)$ and $V_I^c(\tau)$ are two independent real standard Brownian bridges. As a result, we have

$$\frac{1}{T^2} \sum_{t=1}^T \tilde{P}_t \tilde{P}_t^{\bar{}} \rightarrow_d \frac{\omega_e^2}{2} \int_0^1 [(V_R^c)^2(\tau) + (V_I^c)^2(\tau)] d\tau, \quad (14)$$

since $\tilde{\omega}_e^2(l)$ is a convergent estimate of ω_e^2 , we can conclude that

$$\eta^{(\pm i)} \rightarrow_d \frac{1}{2} \int_0^1 [(V_R^c)^2(\tau) + (V_I^c)^2(\tau)] d\tau, \quad (15)$$

as it is claimed.

Remark. Asymptotically, $\eta^{(-1)}$ has the Cramer-von Mises distribution (CvM) under the null hypothesis while the limit theory of $\eta^{(\pm i)}$ ended up as a function of a generalized Cramer-von Mises with two degrees. More precisely, we have $\eta^{(\pm i)} \rightarrow_d \frac{1}{2} \text{CvM}(2)$. The critical values of the seasonal KPSS test with seasonal dummies can be deduced from Nyblom (1989) or Canova and Hansen (1995). These critical values are shown in Table 1.

Table 1 critical values of the seasonal KPSS test

| | 1% | 5% | 10% |
|---------------|-------|-------|--------|
| Root -1 | 0.743 | 0.461 | 0.347 |
| Roots $\pm i$ | 0.537 | 0.374 | 0.3035 |

4 Monte Carlo Simulation

To assess the size properties of the seasonal KPSS statistic, we conduct a Monte Carlo simulation experiment with the seasonal roots of a quarterly process. The data generating process (DGP) for the negative unit root is

$$y_t = x_t' \beta + r_t, \quad t = 1, \dots, T, \quad (16a)$$

with $x_t' \beta$ is defined as in (1) and the autoregressive process r_t ,

$$r_t = \alpha r_{t-1} + v_t, \quad (16b)$$

is driven by normally distributed errors v_t with zero mean and variance one.

The DGP for complex unit roots is given by

$$y_t = x_t' \lambda + c_t, \quad t = 1, \dots, T, \quad (17a)$$

where $x_t' \lambda$ is defined as in (3) and the process c_t is given by

$$c_t = \alpha c_{t-2} + \varepsilon_t, \quad (17b)$$

with the errors ε_t are normally distributed with zero mean and variance 1.

We choose alternative values of $\alpha \in \{-1, -0.8, -0.2, 0, 0.2, 0.8\}$ and we will only consider the nominal size of 5%. Three choices of bandwidth are used, $l0 = 0$, $l4 = \text{integer} \left[4(T/100)^{1/4} \right]$ and $l12 = \text{integer} \left[12(T/100)^{1/4} \right]$. We used 20000 replications and all simulations were carried out using Matlab.

Table 2: Rejection frequencies for the seasonal KPSS test with seasonal dummies for seasonal quarterly unit roots

| α | T | $\eta^{(-1)}$ | | | $\eta^{(\pm i)}$ | | |
|----------|-----|---------------|--------|--------|------------------|--------|--------|
| | | l0 | l4 | l12 | l0 | l4 | l12 |
| -1 | 80 | 0.99 | 0.9999 | 0.9999 | 0.9969 | 1 | 1 |
| | 200 | 0.9997 | 1 | 1 | 1 | 1 | 1 |
| -0.9 | 80 | 0.9229 | 0.9990 | 0.9996 | 0.9736 | 0.9998 | 0.9999 |
| | 200 | 0.9700 | 1 | 1 | 0.9973 | 1 | 1 |
| -0.2 | 80 | 0.1359 | 0.2446 | 0.3396 | 0.1598 | 0.2855 | 0.4382 |
| | 200 | 0.1262 | 0.2427 | 0.2961 | 0.1645 | 0.31 | 0.4165 |
| 0 | 80 | 0.0543 | 0.0678 | 0.1207 | 0.0505 | 0.0775 | 0.1550 |
| | 200 | 0.0526 | 0.0609 | 0.0848 | 0.0508 | 0.0662 | 0.1021 |
| 0.2 | 80 | 0.0183 | 0.0106 | 0.0254 | 0.0112 | 0.0135 | 0.0349 |
| | 200 | 0.0144 | 0.005 | 0.0102 | 0.0086 | 0.0056 | 0.0096 |
| 0.9 | 80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 200 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

The results in Table 2 show that decreasing values of α increase the test size. We also notice that larger data samples do not significantly affect the test size. Our simulation raised another interesting feature, as addressed by Lyhagen (2006) for the case with no deterministic terms, summarized by the fact that in contrast to the ordinary KPSS framework, $I4$ and $I12$ do not have, for the most part, a better size performance than $I0$. In fact in the seasonal KPSS framework, the bad size properties induce substantial power.

Furthermore, the results of Table 1 suggest an overall good power performance of the seasonal KPSS test, particularly against near seasonal unit root alternatives.

4 Conclusion

The joint use of unit root and stationarity tests is advised in empirical studies. Such a joint use can lead to a more rigorous analysis of the dynamic properties of time series. In this paper, we established the asymptotic theory of the seasonal KPSS test in the presence of seasonal dummies, and we then obtained a new test for deterministic seasonality. Considering the problems of seasonal unit root tests power in moderated samples, our test may constitute an adequate solution as it is proved by our simulation study. Lyhagen (2006) also showed its good properties of power when there is no deterministic term in the model. However, the properties of both size and power will be appreciated more in the case of the presence of factors influencing the time series such as measurement errors and additive outliers. It was shown by Khédhiri and El Montasser (2009), through a simulation study that the seasonal KPSS test is robust to the magnitude and the number of additive outliers. Furthermore, the statistical results obtained cast an overall good performance of the test finite-sample properties.

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