Stock-Market Prices with Forgetful Investors

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Abstract

This note investigates the impact of investors' memory limitations on stock-market prices. I consider a simple asset-pricing model in which investors allocate limited cognitive resources to retrieve information from memory and to learn about the data-generating process of multiple assets. I show that the proposed framework may shed light into a number of empirical 'anomalies', including the excess volatility and excess premium of risky assets and the lower return and volatility of more familiar (e.g. large size) stocks.

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I. INTRODUCTION

A large body of research has been devoted to study portfolio decisions under estimation risk.[2] In that literature, it is generally assumed that investors behave as econometricians, applying Bayesian principles to account for the additional risk brought by a limited, but complete, history of observations. In a recent paper, Nocetti (2006) argued that such assumption is restrictive in an important way, namely, that a representative investor's database, i.e., her memory, is likely to depart systematically from that of a statistician. As he showed there, considering memory retrieval limitations as a source of estimation risk has two important implications. First, since the information set retrieved is possibly small, parameter uncertainty remains significant even if the *available* dataset is large and there are no structural shifts. Second, the endogeneity of the information set allows quantifying the magnitude and disentangling the determinants of the deviations from the canonical models of portfolio choice without parameter uncertainty.

This note extends the portfolio allocation problem considered in the abovementioned paper to a simple general equilibrium setting. In particular, I incorporate two new elements to a standard asset-pricing model. First, a constraint in the cognitive resources available to learn about the data-generating process of multiple assets. Second, a technology that transforms those scarce cognitive resources into retrieval of memory traces. Beyond characterizing asset prices when investors are forgetful, I show that the proposed framework may shed light into a number of empirical 'anomalies', including the excess volatility and excess premium of risky assets and the lower return and volatility of more familiar (e.g. large size) stocks.

The note proceeds as follows. Section II presents the structure of the economy, describes the standard portfolio selection problem under estimation risk, presents the optimal portfolio and

[2] Klein and Bawa (1976) is the canonical work in the subject. Closest to this note is Stanbaugh's (1997) analysis of estimation risk under different sample sizes of the assets' returns and Lewellen and Shanken's (2003) analysis of asset pricing under estimation risk.

the optimal policy of memory retrieval under limited cognitive resources, and derives the equilibrium asset prices. In section III I present the main implications of the model and section IV concludes.

II. THE MODEL

The basic economy that I analyze is the same as that in Lewellen and Shanken (2003), which is based on DeLong et al. (1990). Individuals live for two periods and the resources they have to invest are exogenous. The economy contains a riskless asset in perfectly elastic supply that pays a real dividend *r* and *m* risky assets, with one unit outstanding each, which pay a real dividend D_t (an $mx1$ vector) and have an $mx1$ price vector P_t.

Define the vector of gross excess returns as $R_{t+1} \equiv P_{t+1} + D_{t+1} - (1+r)P_t$ and suppose that dividends are i.i.d. multivariate normal with mean vector μ and known covariance matrix Σ. The representative investor does not known μ and has to estimate it using past data. The optimal portfolio with estimation risk is obtained by maximizing expected utility under the predictive distribution of returns (see, e.g., Klein and Bawa 1976),

$$\begin{aligned} w &= \arg\max_w \int_{R_{t+1}} U(w) p\left(R_{t+1} \middle| \Theta_t\right) dR_{t+1} \\ &= \arg\max_w \int_{R_{t+1}} \int_{\mu} U(w) p\left(R_{t+1}, \mu \middle| \Theta_t\right) d\mu dR_{t+1} \end{aligned} \tag{1}$$

where $U(w)$ is the utility function, $p\left(R_{t+1} \middle| \Theta_t\right)$ is the predictive density and $p\left(R_{t+1}, \mu \middle| \Theta_t\right) = p\left(R_{t+1} \middle| \mu, \Theta_t\right) p\left(\mu \middle| \Theta_t\right)$ is the posterior density of μ. Therefore, the Bayesian solution maximizes expected utility over the distribution of the parameters.

As I shall demonstrate, the allocation of cognitive resources affects portfolio shares by changing the predictive density of excess returns. Before analyzing the model with limited cognitive resources, however, I briefly consider the solution to (1) for two benchmarks: the case with no estimation risk and that of exogenous estimation risk.

I assume that the representative investor has CARA preferences [i.e. $U = -\exp(-\gamma W)$] and, for simplicity, that dividends are uncorrelated. Then, in a rational equilibrium without estimation risk, where prices are constant, the optimal share in asset *i* is

$$w_i = \frac{\mu_i - rP_i}{\gamma \sigma_i^2}, \qquad (2)$$

where μ_i and σ_i^2 are the mean and the variance of asset *i*'s dividends, and P_i is the price of the asset.

Now consider the case with estimation risk and an exogenous number of observations n_i for asset *i*. Under a diffuse prior of the dividend process, its posterior (subjective) distribution is $D_{t+1} \sim N(\hat{\mu}_t, \Lambda)$, where $\hat{\mu}_t$ is the sample mean vector and Λ is the diagonal covariance matrix with elements $\sigma_i^2(1 + 1/n_i)$. I show later that, in equilibrium, estimation risk introduces price volatility. Moreover, I show that price volatility is proportional to the dividends sample variance such that $Var_t^*(R_{t+1})$ -the subjective variance of returns- has diagonal elements $\sigma_i^2\left(1 + (1/n_i)(1 + c)\right)$, where *c* is a constant and identical for all assets.

Therefore, under estimation risk, $Var_t^*(R_{t+1})$ can be decomposed into three elements: dividends volatility (σ_i^2), dividends estimation risk $\sigma_i^2(1/n_i)$, and price volatility $\sigma_i^2(1/n_i)c$. As a result of the higher return volatility, the Bayesian investor selects a portfolio with less risk. Alternatively, in equilibrium, assets convey a higher risk premium. However, as is clear from the above decomposition, as $n \to \infty$ the effect of estimation risk disappears.

II.1. Memory retrieval mechanism

I imagine a representative investor with a stock of memories of the entire history of dividends that relies on the *retrieval* of a subset of those memories to learn about μ.[3] Following Anderson's (1989, 1993) ACT-R (Adaptive Control of Thought-Rational) theory, the mechanism of memory retrieval is conceptualized as an optimization of the task that the cognitive system faces. Specifically, according to ACT-R, given a query about one or more past target events, the cognitive system searches and retrieves information previously stored in the memory database until the expected value of the to-be-remembered evidence is lower than its expected cost (the cost of the effort or time exerted by the cognitive system to retrieve a memory trace).

To formalize this retrieval procedure within the present framework, I assume that a representative investor is endowed with a limited amount *k* of mental energy resources, which is devoted to learn about the dividends' data-generation processes. In particular, denoting e_i as the energy allocated to searching and retrieving information about asset *i*, the resource constraint is $\sum_{i=1}^{m} e_i = k$. I make two additional assumptions: First, the information-search process for a given asset becomes increasingly difficult (i.e. more mental energy is required) as more information is retrieved; Second, the cognitive system is relatively more productive, in terms of mental energy exerted, searching and retrieving information about more familiar stocks.[4] Specifically, I propose that a *cognition technology* transforms mental energy into information retrieval about asset *i* as follows

[3] This is consistent with evidence (e.g. Shefrin, 2005 p. 264-274) that investors tend to rely in their memory to form return expectations.

[4] Schneider and Chein (2003) provide a review of experimental and neuropsychological evidence on the relation between information processing and familiarity.

$$\Phi_i e_i^{\alpha} = n_i, \quad (3)$$

where $\alpha < 1$ and Φ_i is a parameter that measures familiarity.[5]

Given this technology and the resource constraint we obtain the *cognition/memory possibilities set*

$$\sum_{i=1}^{m} \left(\frac{n_i}{\Phi_i} \right)^{\frac{1}{\alpha}} = k, \quad (4)$$

which represents all the possible combinations of memory output for a given level of productive resources.

II.2. Portfolio selection with memory limitations

The portfolio selection problem proceeds in two steps [see Muendler (2003) for a similar characterization]: First, given an arbitrary prior belief of returns for asset *i*, say $E_0^*\left(R_{i,t+1}\right) \; \forall i$ (recall the assumption of diffuse priors), the cognitive system selects the allocation of energy resources that minimizes estimation risk (i.e. the portfolio variance); Second, given the information retrieved, the representative investor calculates the conditional estimate of excess returns, $E_t^*\left(R_{i,t+1}\right)$, and selects the optimal portfolio shares.

Formally, using the fact that $Var_t^*\left(R_{t+1}\right)$ has diagonal elements $\sigma_i^2\left(1+\left(1/n_i\right)(1+c)\right)$, the first order condition for the (*ex-ante*) portfolio share of asset *i* is

$$E_0^*\left(R_{i,t+1}\right) - \gamma\sigma_i^2\left(1+\frac{1}{n_i}(1+c)\right)w_i = 0 \quad (5)$$

[5] The interpretation of equation (3) is that the investor recalls n_i realizations of the dividends' process for asset i. Alternatively, one can think about the investor as recalling "news" reports about asset i that are perfectly correlated with the dividends process.

and the first order condition for the cognition problem of asset *i* is

$$\frac{\gamma w_i^2 \sigma_i^2 (1+c)}{2n_i^2} = \lambda \frac{1}{\alpha}\left(\frac{n_i}{\Phi_i}\right)^{\frac{1-\alpha}{\alpha}} \frac{1}{\Phi_i} 0, \tag{6}$$

where λ is the Lagrange multiplier of the cognitive resource constraint. Just as the ACT-R theory proposes, equation (6) asserts that the cognitive system retrieves information until the expected benefit of the marginal piece of information, a decrease in the subjective variance of excess returns, equals its marginal cost.

Denoting n_i^* as the resulting optimal sample size, the (conditional) portfolio share in asset *i* is

$$w_i = \frac{E_t^*\left(R_{i,t+1}\right)}{\sigma_i^2 \theta_i}, \tag{7}$$

where $\theta_i \equiv \gamma\left[1+\left(1/n_i^*\right)(1+c)\right]$ represents the *effective* degree of risk aversion of the investor with bounded memory.

II.3. Equilibrium

Applying the market clearing condition (i.e. the demand for the risky assets equals their supply, $w_i = 1\ \forall$i) to the cognition allocation and the portfolio selection policies leads to

Proposition 1. *The equilibrium amount of information retrieved for asset i (i=1,…,m) is*

$$n_i^* = \frac{k^\alpha \Phi_i}{1+\left(1/\sigma_i^2\Phi_i\right)^{\frac{1}{1+\alpha}} \sum_{\in v \neq i}\left(\sigma_v^2/\Phi_v\right)^{\frac{1}{1+\alpha}}}, \tag{8}$$

and the price of asset i (i=1,…,m) is

$$P_{i,t} = \frac{1}{r}\hat{\mu}_{i,t} - \frac{1}{r}\sigma_i^2\gamma\left[1+\frac{1}{n_i^*}\left(1+\frac{1}{r^2}\right)\right]. \qquad (9)$$

Proof. See appendix.

The price of asset *i (i=1,…,m)* is identical to that in the Bayesian case with estimation risk, except, of course, that the information set is endogenous.

III. ANALYSIS

In the following subsections I discuss several implications of the equilibrium conditions (8) and (9). For ease of exposition, I assume that the available information would permit calculating the parameters without error (i.e. the cases of no estimation risk, exogenous estimation risk, and of an infinite endowment of mental energy are identical).

III.1. The Level of Stock Prices and 'Size Effects'

As in the case with no estimation risk, the price of a risky asset is the expected present value of dividends adjusted by a risk factor. Asset prices, however, are lower in our model. The reason is simple. The investor with limited cognitive resources, who has larger effective risk aversion, requires a larger return on equity and therefore, in equilibrium, asset prices are lower.

The endogeneity of estimation risk permits disentangling the determinants of the difference in the level of prices relative to the case with no such risk. A larger endowment of mental energy resources reduces the difference in the level of prices since it allows the representative investor to retrieve more information. Everything else constant, a larger number of assets in the portfolio decreases the level of prices relative to the standard case since the limited cognitive resources have to be divided among more tasks. Also, in equilibrium, more energy is allocated to recall information about assets that are more volatile and, therefore, their prices are closer to the standard model.

Finally, inspection of optimal memory retrieval in (8) shows that the amount of information recalled for asset *i* increases more than proportionally with the familiarity/productivity of this asset than with the familiarity/productivity of the other assets. That is, an increase in productivity produces a biased expansion of the cognition possibilities set and makes it optimal to increase the allocation of energy to the now relatively more energy-intensive asset. Thus, in equilibrium, more familiar assets have lower expected returns and higher prices. If, as one would expect, familiarity is linked to firm size, the model is able to replicate the well know regularity that expected returns are negatively related to size (e.g. Banz, 1981; Fama and French, 1992). This result corroborates Fama and French's intuition that firm size might be a proxy for risk.

III.2. Prices and "fundamental" risk

How does the endogenous characteristic of estimation risk alter the comparative static analysis of the dividends' volatility? In the benchmark case, prices are a linear function of the dividends' volatility,

$$\left(\frac{\partial P_i}{\partial \sigma_i^2}\right) = -\frac{1}{r}\gamma . \tag{10}$$

By endogenizing effective risk aversion, the effect of "fundamental" risk on prices is no longer linear. In fact, the change in prices is given by

$$\left(\frac{\partial P_i}{\partial \sigma_i^2}\right) = -\frac{1}{r}\left(\theta + \sigma_i^2 \frac{\partial \theta_i}{\partial n_i^*}\frac{\partial n_i^*}{\partial \sigma_i^2}\right)$$
$$= -\frac{1}{r}\gamma\left[1+\frac{1}{n_i^*}\left(1+\frac{1}{r^2}\right)\left(1-\frac{\sigma_i^2}{n_i^*}\frac{\partial n_i^*}{\partial \sigma_i^2}\right)\right]. \tag{11}$$

From equation (7) it follows that more cognitive resources are allocated to asset *i* when its fundamental volatility increases, $\left(\partial n_i^* / \partial \sigma_i^2\right) > 0$. This, in equilibrium, diminishes the effect of risk on asset prices since the marginal benefit of retrieving information about this asset is greater.

The comparative static analysis gives us another interesting implication. Because the assets are uncorrelated, in the case with no estimation risk the equilibrium price of each asset is completely unrelated to the process of the other assets. In contrast, in the current scenario the variance of asset *j*'s dividends is involved in the determination of the equilibrium price of asset *i* since the information retrieved is produced from a common pool of cognitive resources. In particular, we have $\partial P_i / \partial \sigma_j^2 < 0$; an increase in the dividends' variance of one asset reduces the incentive to retrieve information about other assets, reducing their prices.

III.3. The Risk Premium

What are the implications of memory deficits for the risk premium? Consider the case of *m* identical risky assets, which implies that the memory sample size and the portfolio shares are the same for each asset. Then, it follows that the premium on the risky assets is

$$\gamma Var_t^*\left(R_{i,t+1}\right) = \gamma\sigma_i^2\left[1+\frac{1}{n_i^*}\left(1+\frac{1}{r^2}\right)\right] = \gamma\sigma_i^2\left[1+\left(\frac{m}{k}\right)^{\alpha}\frac{1}{\Phi}\left(1+\frac{1}{r^2}\right)\right]. \tag{12}$$

This yields,

Proposition 2. *In an economy with m identical risky assets the risk premium is larger than the case with no estimation risk by the amount* $\gamma\sigma^2\left(\frac{m}{k}\right)^{\alpha}\frac{1}{\Phi}\left(1+\frac{1}{r^2}\right)$.

In the standard Bayesian framework, the excess premium due to estimation risk disappears as the *available* information increases. The proposition establishes that, due to scarce cognitive resources, the excess premium may remain significant even if the data available is large and there are no structural shifts.

The comparative statics are straightforward. The premium decreases if the representative investor has a larger endowment of cognitive resources or if she becomes more productive at retrieving information from memory. If the investor needs to divide her limited resources over a larger number of firms the parameter's precision will be lower and estimation risk higher. Finally, if the investor becomes more risk averse or the dividends' volatility increases she will require an excess premium to bear the additional subjective risk.

Whether the model can produce quantitative results that resolve the equity premium puzzle (Mehra and Prescott, 1985) depends on the assumptions one wants to make about the cognition technology. What is important to note, however, is that the estimation risk premium can be substantial. For example, for a real interest rate of three percent and $n \in [10,50]$ for all assets, the model implies that effective risk aversion is in the range of 20 to 110 times the value of the coefficient of absolute risk aversion.

III.4. Excess Volatility of Stock Prices

Since the seminal works of Shiller (1981) and Leroy and Porter (1981) much research has been devoted to study the excess volatility of stock prices. Shiller (1981) argued that in a stationary framework with rational expectations the variance of the ex-post rational price (using actual dividends) must be at least as large as the variance of actual prices (based on expected dividends). This establishes a volatility bound which clearly does not hold in empirical work.

As argued by Lewellen and Shanken (2003), estimation risk might provide an answer to the observed regularity. Specifically, the variance of stock prices based on realized dividends is

$$Var\left[\sum_{k=1}^{\infty}\frac{1}{(1+r)^k}D_{i,t+k}\right]=\frac{1}{r^2+2r}\sigma_i^2, \tag{13}$$

while, from (9) the variance of realized prices is $\left(1/r^2 n_i^*\right)\sigma_i^2$. Therefore, the volatility bound is violated if $n_i^* \leq 1+2/r$.

In contrast to previous studies, the endogeneity of estimation risk in the present framework provides some insight as to the determinants of the excess volatility. Since the risk premium is proportional to price volatility we have the same comparative static analysis (see subsection III.3). A point worth reiterating is that an increase in the familiarity of asset *i* reduces the price volatility of this asset and increases the volatility of the other assets. There is some evidence that supports this result. For example, if firm size is representative for memory retrieval productivity due to familiarity of the asset, Brown and Ferreira (2004) show that smaller firms have higher excess volatility. Additional evidence is provided by Pastor and Varonesi (2003) and Brown and Ferreira (2004) who find that idiosyncratic volatility is negatively related to firm age. We should expect that newer firms are less familiar to the average investor.

IV. CONCLUSIONS

This note presented a simple model that integrates an economic-psychological theory of memory retrieval with a Bayesian model of estimation risk to explain a number of stylized facts in asset-pricing that are difficult to reconcile with the standard assumption that investors use all available information to form return expectations. The results derived, however, might seem to rely heavily on a number of assumptions that I made. For instance, I assumed the existence of a representative investor who faces quite stringent mental constraints. In most stock markets, however, professional investors represent the largest fraction of trade and they, arguably, behave in a 'Standard Bayesian' manner using all available information. Although Shefrin (2005) presents evidence that professional investors indeed rely on their memory to form expectations, allowing for heterogeneous agents can be readily addressed as in De Long et al. (1990) model of sophisticated and noise traders. Since the setup in this paper is the same as that in De Long et al. I expect the results to be qualitatively robust to heterogeneity. Quantitatively, however, the effects would depend on the fraction of traders that have limited memory.

Also, given the assumption of diffuse priors, memory deficits do not affect expected returns. Suppose, instead, that the prior belief about the dividend process of asset *i* is normal with mean μ_{i0} and variance σ_i^2/ϕ_i, where ϕ_i determines the prior precision. With this prior, and again assuming that the assets are uncorrelated, a Bayesian investor's belief about dividends of asset *i* is

$$D_{i,t+1}\big|\sigma_i^2 \sim N\left(\frac{\phi_i}{n_i+\phi_i}\mu_{i0}+\frac{n_i}{n_i+\phi_i}\hat{\mu}_{i,t+1},\sigma_i^2\left(1+\frac{1}{n_i+\phi}\right)\right).$$

Expected dividends are now a weighted average of the prior and the sample estimates. It is straightforward (e.g. Lewellen and Shanken, 2003) that prices will take the same form as in (9). Compared to the case of diffuse priors, however, prices are less volatile and the risk premium is smaller. The reason is simple. Since investors shrink their estimates toward their prior beliefs, which by assumption have a constant mean, the variance of expected dividends is smaller.

Finally, although I have followed most of the literature dealing with estimation risk in considering the case of i.i.d. dividends, the results presented here also hold in non-i.i.d. settings. For instance, Kandel and Stambaugh (1996) show that when returns are predictable (follow an AR(1) process) an investor that faces parameter uncertainty should use all available observations to form expectations. Importantly, they present explicit solutions for the predictive variance of returns and show that it decreases with the sample size. Therefore, the results presented in this paper could be easily generalized to this setting.

APPENDIX

Proof of proposition 1.

Equilibrium is such that the representative agent follows the optimal policies (5) and (6) and the demand for the risky assets equals their supply, $w_i = 1\ \forall \mathrm{i}$. The optimal allocation of attention follows directly by using the market clearing condition in equation (5) and the resource constraint in (4).

Also, market clearing implies that

$$E_t^*\left(P_{i,t+1} + D_{i,t+1}\right) - \left(1+r\right) P_{i,t} = \gamma Var_t^*\left(P_{i,t+1} + D_{i,t+1}\right).$$

Solving for the price of asset *i* at time *t* yields

$$P_{i,t} = \frac{1}{1+r}\left[E_t^*\left(P_{i,t+1} + D_{i,t+1}\right) - \gamma Var_t^*\left(P_{i,t+1} + D_{i,t+1}\right)\right].$$

Without any lack of generality I drop the subscripts that denote the asset. In order to solve for the equilibrium asset price guess that the solution to the difference equation $P_t = \frac{1}{1+r}\left[E_t^*\left(P_{t+1} + D_{t+1}\right) - \gamma Var_t^*\left(P_{t+1} + D_{t+1}\right)\right]$ is $P_t = A + \alpha\hat{\mu}_t$. Substituting this guess into the pricing function we have $A + \alpha\hat{\mu}_t = \frac{1}{1+r}\left[E_t^*\left(A + \alpha\hat{\mu}_{t+1} + D_{t+1}\right) - \gamma Var_t^*\left(A + \alpha\hat{\mu}_{t+1} + D_{t+1}\right)\right]$.

The best guess for the expected future estimate and future dividends is the current estimate. Therefore, $E_t^*\left(A+\alpha\hat{\mu}_{t+1}+D_{t+1}\right)=A+(\alpha+1)\hat{\mu}_t$. We also know that the variance of the estimate is σ^2/n and the subjective variance of dividends is $\sigma^2\left(1+\left(1/n\right)\right)$. Substituting these results into the pricing function and equating coefficients leads to the relationship in the text.

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