THE 2000 TURKISH CELL-PHONE LICENSE AUCTION*

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Abstract

The Turkish government intended on selling two cell phone licenses in 2000 with an unusual auction design. Specifically, the design was such that the winning bid in the first auction would be the reserve price in the second auction. We show that such an auction design gives predatory bidding incentives which will result in only one license being sold. We compare this auction design with the other well-known auction designs which allow the government to sell two licenses. Depending on the (marginal) cost difference of the bidding firms, the Turkish auction design may generate more revenue. Further, we discuss the social welfare implications. Finally, we show that one can design an auction (sometimes selling only one license) that will give better or same results than that of the Turkish auction.

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1 Introduction

The Turkish government intended on selling two cell-phone licenses with a sequential auction in 2000. Its design was very different than from those generally used before. Specifically, the winning bid in the first auction would be the reserve price in the second auction. We show that the Turkish auction design gives predatory (preemptive) bidding incentives which results in only one license being sold, which was in fact the case in the 2000 Turkish auction. However, depending on the cost asymmetry of the bidding firms, this may result in higher seller revenue or (even) higher social welfare compared to some other auction designs that result in two licenses being sold.

In 2000, two incumbent firms were already operating in the Turkish cell-phone market. The government announced that two more licenses would be sold with the auction rules as defined above.[1] The incumbent firms were not allowed to participate in the auctions. Six groups/firms participated in the first auction. A Turkish-Italian group, Is-Tim, won the first license by bidding an amount much higher than the analysts' expectations. This "high" bid became the reserve price in the second auction. No firm participated in the second auction since benefit of the second license (profits) would not justify the cost (paying the second auction's "high" reserve price). Is-Tim was therefore able to deter the entry of another firm by strategically bidding. Turkish government, fearing law suits or reputation concerns, did not try to auction the unsold license later.

In our model, one of the bidder firms (an efficient firm) has the same cost function as the incumbent monopolist.[2] The other bidders are identical high-cost (inefficient) firms. All costs are common knowledge.[3] The value of each license depends on which firms will receive the licenses and the resulting market structure; hence, the license valuations are endogenous.[4]

[1] A third license was reserved for the state-owned telecommunications company.

[2] We make some small variations such as modelling one incumbent firm instead of two; however, the number of incumbent firms will not change the qualitative results as long as there is room for at least one entrant in the downstream market. We also do not model the fact that a third license is reserved for the state-owned company. This does not affect the qualitative results.

[3] Complete information auction models are used in papers such as Krishna (1993 and 1999), Jehiel and Moldavanu (1996), Grimm, Riedel, and Wolfstetter (2003), Riedel and Wolfstetter (2006), and Hoppe, Jehiel, and Moldavanu (2006). This assumption rules out complexities like the winner's curse and enable us to focus on the link of auctions with revenue and market structure.

[4] Some of the complete information papers that study endogenous license valuations are Krishna (1993,

We show that the low cost firm successfully deters the entry of another firm by bidding predatorily. We also show that if the government were to use a simultaneous auction (or a sequential auction) with no reserve price, then it would be able to sell two licenses. We compare the Turkish auction with these auctions. First, we find that the seller may receive higher revenue with the Turkish auction. Second, we find a non-monotonic relation between the bidders' cost asymmetry and the seller's revenue. Third, we discuss the social welfare implications. Finally, we show that one can always design an auction, which some of them involves selling one license, that will give better or same results as the Turkish auction.

Krishna (1993, 1999), Rodriguez (2002), and Hoppe, Jehiel, and Moldavanu (2006) are some other papers that analyze the preemptive bidding incentives.[5] These aforementioned papers do not study the Turkish auction design.

Klemperer (2002) writes that the Turkish GSM auction is biased towards creating monopoly without modelling the details. We show that his insight is correct. Moreover, we compare this auction with the other well-known auctions that will allow the government to sell two licenses and show when the Turkish auction will generate more revenue.

There are many papers that study the real-world auctions. For example, Grimm, Riedel, and Wolfstetter (2003), and Hoppe, Jehiel, and Moldavanu (2006) analyze German spectrum auctions. However, -to our best knowledge- there is only one published paper, Gunay and Meng (2007), that study the Turkish auction in the literature. Another exception is Ozcan's (2004) working paper. Unlike these two papers, our modelling of cost function enables us to show the link between the bidders' cost asymmetry and the seller's revenue. We find a non-monotonic relation. We compare the Turkish auction with the other well-known auction designs that result in selling two licenses. We show that the Turkish auction may generate more revenue.[6] Unlike these two papers, we show that the Turkish auction design may generate higher social welfare for some parameter spaces compared to a simultaneous

1999), Rodriguez (2002), Hoppe, Jehiel and Moldovanu (2006). Some of the incomplete information papers are Rosenthal and Wang (1996) and Ozcan (2004).

[5]The literature uses the term preemptive bidding when the incumbent bids in order to prevent the new entry. Here, we use the term predatory bidding since not the incumbent, but one of the entrants is bidding (predatorily) to prevent the entry of others.

[6]In his working paper, Ozcan compares the Turkish auction only with selling monopoly rights. We think that the comparison should be done with the auctions that result in selling two licenses.

auction that sells two licenses.

In short, our modest aim is to model the 2000 Turkish cell-phone license auction and compare it with the other auction designs. In what follows, we set up our simple model, and discuss whether and when to use the Turkish auction design at the end.

2 The Model

We assume that the government will sell two cell-phone licenses in the form of a sequential auction administered in the same period. Each auction format is a first-price sealed-bid auction. Each entrant firm can buy at most one license.

There is already one incumbent firm in the market which cannot participate in the auction. The incumbent firm and the successful entrants will Cournot-compete at each period $t = 0, 1, ...$ In the downstream market, the inverse demand function is $p(q) = A - bq$, where $p > 0$ and $q \geq 0$ denote the market price and the market quantity demanded, respectively, and the parameters A and b are positive. There are at least 3 bidding firms. One of the bidding firms and the incumbent firm have a zero marginal cost; these low cost firms are denoted as firm L. The other bidding firms are identical to each other and has a marginal cost, $c > 0$. Each of these firms is denoted as firm H.[7] All firms have zero fixed costs. Each firm will decide how much to bid in the first and in the second auction, if it participates.

We will look for a Subgame Perfect Nash equilibrium where symmetric bidders use symmetric strategies and do not use (weakly) dominated strategies. The bids will be multiples of a very small monetary unit ϵ.[8] If there is a tie between n firms, one of these firms wins with some positive probability, $\kappa = \frac{1}{n}$. We denote profit with π^i_j, where $i = L, H$ shows the firm type and $j = LL, LH, LLH, LHH$ shows the number and type of firms competing in the market. For example, π^H_{LHH} denotes the profit of firm H when there is a total of three firm in the market (one L type and two H types).

[7] We can assume that these H-firms are not identical. The qualitative results will not change.

[8] For notational simplicity, we assume that all profits will be multiples of ϵ. This monetary unit ϵ is almost equal to zero.

2.1 The Turkish Auction

In this auction, the winning bid of the first auction will be the reserve price for the second auction; that is, the firm who wins the second auction will pay **strictly more** than the first auction's winning bid.

Assumption 1: A firm will not participate in the second auction if it expects a negative payoff (after paying the reservation price.)

First we show that there are many weakly dominated strategies. Let $(s_{i1}, s_{i2}) \in S_i$, where s_{i1} and s_{i2} denote the firm i's action, $i = L, H$, in the first and second auction, respectively. S_i is the strategy space of firm i. Note that $s_{i2} \in \{0, \epsilon, 2\epsilon ...\} \cup \{\textit{not participate}\}$ due to assumption 1.

Lemma 1 *: Let there be one L, and two H bidders. Moreover assume that the bidders will not participate in the second auction if they will make negative profits after paying the reserve price. For H-type firms and* $j = LH, LLH$*, the strategy* $(\pi_j^H - \epsilon, s_{H2}) \in S_i$ *weakly dominates* $(\pi_j^H + k\epsilon, s_{H2}) \in S_i$*, where* $k = 0, 1, 2, ...$*.*

This lemma essentially tells that a strategy that involves an action in which a high type firm bids more than (or equal to) its "expected" profits in the first auction is weakly dominated. According to our equilibrium concept, strategies involving such actions cannot be an equilibrium.

In the proposition below, we show that no firm will participate in the second auction. Firm L wants less competition in the market, so it will bid predatorily in the first auction to make sure that firm H will not participate in the second auction. Firm L bids slightly higher than firm H's highest possible bid and will win the first auction. Let us emphasize that the bid of L firm depends on the cost asymmetry between the firms.[9]

Proposition 2 *(Equilibrium in the Turkish auction)*

A)If $0 < c < \frac{A}{11}$*, then firm L will win the first auction by bidding* $\pi_{LH}^H = \frac{(A-2c)^2}{9b(1-\delta)}$*. No firms will participate in the second auction.*

[9] Note that c is not only the marginal cost of H type firm but also the cost difference between L and H type firm.

B)If $\frac{A}{11} \leq c \leq \frac{A}{3}$, then firm L will win the first auction by bidding $\pi^H_{LLH} = \frac{(A-3c)^2}{16b(1-\delta)}$. No firms will participate in the second auction.

While firms are bidding, they take into account the possible competition (and the resulting profits) in the downstream market. To understand Proposition 2 part B, assume that one of the H-firms wins the first auction by offering the possible maximum (not weakly dominated) bid that gives them (almost) zero expected profit. Moreover, assume that firm L participates in the second auction. Since the marginal cost of firm H is high in this region, firm L can still make a positive profit after paying the reservation price. But then, in the first auction, when firm H is bidding, it should bid assuming that firm L will also be in the market. This bid will be $\pi^H_{LLH} - \epsilon$ due to competition between the H-firms.[10] But firm L is better off if there is less competition in the market. Hence, firm L will bid π^H_{LLH}, just ϵ higher than H-type firms' bid in the first auction. No firm H can bid more than this (reservation price) without making negative profits, and hence, no firm will participate in the second auction. Firm L's predatory bidding strategy will work.

To understand part A, assume that firm H wins the first auction. Since the cost of firm H is low enough, firm L cannot make enough profits in the downstream market after paying the reservation price in the second auction. That is, firm H deters the entry of firm L. Specifically, firm H will bid $\pi^H_{LH} - \epsilon$ in the first auction as if there will be two firms in the market. Expecting this, firm L will bid slightly more than firm H's bid in the first auction and deter the entry of firm H.

Complete-information endogenous valuation papers such as Gunay and Meng (2007) and Hoppe et. al. (2006) do not allow cost differences among the entrant firms/bidders; however, we show that the cost difference affect the bids (and hence seller's revenue). Note that as c increases, the bids (hence the seller revenue) decrease but there is a discontinuity point.

2.2 "No Reserve Price" Simultaneous Auction

Now, we assume that the government designs a simultaneous auction with no reserve price in which two licenses are for sale. Using first or second sealed bid price format will give

[10]Lemma 1 shows that strategies containing π^H_{LLH} is weakly dominated by strategies containing $\pi^H_{LLH} - \epsilon$.

the same outcome since this is a complete information game. We will call this auction "no reserve price auction." Also, note that this auction's outcome is the same as the outcome of selling two licenses in a sequential auction with no reserve price.

Proposition 3 *(Equilibrium in "no reserve price auction") In a no reserve price auction, two licenses will be sold. Firm L will win by bidding* $\pi^H_{LLH} = \frac{(A-3c)^2}{16b(1-\delta)}$ *and one H firm will win by bidding* $\pi^H_{LLH} - \epsilon$.

Once the reservation price is removed from the auction design, both licenses are sold. This shows that setting the reserve price that can be manipulated with a bid in the first auction causes only one license to be sold. Now, we can compare these two different auctions in terms of seller's revenue and social welfare.

2.3 Comparison Of Two Auctions

Since ϵ is very small (almost zero), we assume that the seller's revenue in "no reserve price auction" is equal to $2\pi^H_{LLH}$. We make this assumption for exposition purposes. Qualitative results do not change because of this assumption.

Proposition 4 *(Revenue Comparison) The government will have more revenue with a no reserve price auction compared to the Turkish auction when*

$$c < \frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} \approx \frac{A}{20} \quad or \quad \frac{A}{11} \leq c \leq \frac{A}{3} \tag{1}$$

The government will have more revenue with the "Turkish auction" when

$$\frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} < c < \frac{A}{11} \tag{2}$$

This Proposition follows from Proposition 2 and 3 directly and confirms that the government will have more revenue from selling one license through the "Turkish auction" when $\frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} < c < \frac{A}{11}$, otherwise, the government will get more revenue from selling two licenses through a no reserve price auction.

As one can see from figure 1, there is not a monotonic relation between the firms' cost difference and the seller's revenue when we compare the two auctions. When $0 < c < \frac{A}{11}$, in a no reserve price auction, government revenue will be $2\pi^H_{LLH}$. In the Turkish auction, the government revenue will be π^H_{LH}. These bids are decreasing functions of the marginal cost (and the cost difference), c. As a result, bids (hence the seller's revenue) decrease in both type of auctions. However, as c increases, government revenue in a no reserve price auction will decrease **faster** than that in the Turkish auction. When c is zero (the extreme case), "no reserve price" auction gives a higher revenue. Since bids will decrease faster in the no reserve price auction, as c increases, eventually, revenue becomes higher in the Turkish auction. This happens when $\frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} < c < \frac{A}{11}$.

At $c = \frac{A}{11}$ there is a discontinuous jump in the Turkish auction case. This is the point in which the market is profitable for three firms rather than two firms. Beyond this point, entry deterrence is much cheaper for the L firm, and it bids π^H_{LLH} in the Turkish auction which is also the seller's revenue. In the no reserve price auction, two licenses are sold for a total of $2\pi^H_{LLH}$. Hence, revenue will be higher with the no reserve price auction.

Proposition 5 *(Social Welfare Comparison)*

A) When $\frac{7A}{69} < c \leq \frac{A}{3}$*, there will be higher social welfare with the original auction than that with a no reserve price auction.*

B) When $0 < c < \frac{7A}{69}$*, there will be higher social welfare with a no reserve price auction.*

If the cost difference between the firms is large enough; that is when $\frac{7A}{69} < c \leq \frac{A}{3}$, then having one firm in the market is better than having two firms in terms of social welfare. In other words, less competition results in higher social welfare. The reason for this result is as follows. When the high-cost firm enters the market, the total output, and hence, consumer surplus increases. This effect increases social welfare. However, the low-cost firm produces fewer outputs in the new equilibrium. The high-cost firm produces outputs

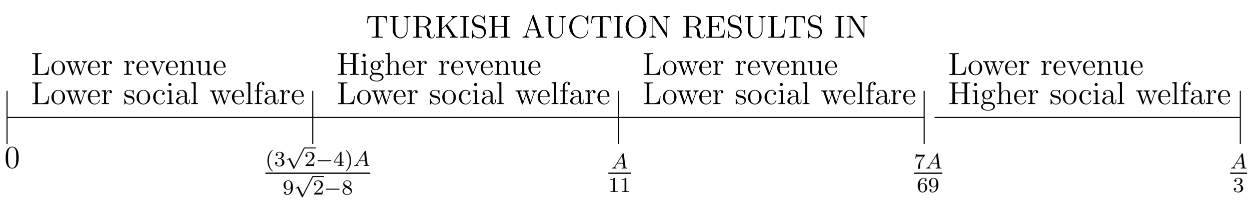

Figure 1: Comparison of Turkish and no reserve price auctions under different cost ranges

previously produced by the low cost firm. As a result, total profits in the new equilibrium will be lower. This decreases the social welfare. When the cost asymmetry between the firms is high, the latter effect dominates and the social welfare decreases.

In Lahiri and Ono (1988) and Zhao (2001), if the less efficient firm's marginal cost decreases, then the social welfare decreases. Since the Turkish auction prevents the entry of the high cost firm, we find that the Turkish auction gives higher social welfare compared to the "no reserve price" auction. However, we caution the reader that this is a complete information model. Hence, if the cost asymmetry were in that region, the government could have auctioned only one license with a no reserve price auction. Because of competition, the seller's revenue would be higher while the social welfare is the same with this auction.

3 When to use the Turkish auction design?

Should we use the Turkish auction? We will show that a seller who cares about social welfare or revenue can always use a different auction design that gives better or same results than the Turkish auction. We have already discussed that a seller who wants to maximize social welfare can do better (or same) by using other auction designs. What about a seller who wants to maximize revenue? Compared to no reserve price auction that sells two licenses, we have showed that the Turkish auction can generate more revenue for some c. However, if c is in such a region, then the Turkish government could sell one license with a no reserve price auction, then, the seller's revenue would be equal to π_{LH}^{H} which is the same as the Turkish auction.[11]

4 Conclusion

In this paper, our modest aim was to analyze the 2000 Turkish cell-phone license auction. We showed how this auction design gives predatory (preemptive) bidding incentives to the bidders.

[11] Depending on the government and firm's discount factors, government may even get more revenue by setting a higher exogenous reserve price. Our aim is to show that one can find other auction designs that perform same or better than the Turkish auction design. Since we showed one, we do not model this possibility.

Our main result is that a seller who cares about maximizing social welfare or revenue can always design another auction that gives better or same results as the Turkish auction.

5 Appendix

Proof of Lemma 1: We will prove this for the case $j = LH$; that is, the case in which the H firm will not participate in the second auction if it loses the first one by Assumption 1 since it will be making negative profits. The proof for the case $j = LHH$ is similar and omitted.

Let S_i be the strategy space of firm $i = H, L$ and σ_i a strategy. We will show that one H firm's strategy $(\pi_{LH}^H + k\epsilon, x) \in S_i$, where $k = 0, 1, 2, ..$, is weakly dominated by the strategy $(\pi_{LH}^H - \epsilon, x) \in S_i$. Let $(s_{i1}, s_{i2}) \in S_i$ be the strategy of the $i = L, H$ firm. The first and second components of the strategy show the action in the first and the second auction, respectively. In short, we have to show that the payoff of the H-type firm $U(.)$ is lower under weakly dominated strategies:

$$U((\pi_{LH}^H + k\epsilon, x), \sigma_H, \sigma_L) \leq U((\pi_{LH}^H - \epsilon, x), \sigma_H, \sigma_L) \tag{3}$$

The relation should hold with strict inequality at least for one strategy combination. We will show this for different cases.

Case 1: $Max\{s_{H1}, s_{L1}\} \geq \pi_{LH}^H + k\epsilon$.

In this case, both strategies give zero payoff since the H-firm will lose the first auction and will not participate in the second auction according to our assumption. They will not participate since then they will be the third firm and their profits will definitely be lower than the reserve price of the second auction.

Case 2: $\pi_{LH}^H - \epsilon < Max\{s_{H1}, s_{L1}\} \leq \pi_{LH}^H + k\epsilon$

In this case, the left hand side of inequality 3 is non-positive (and negative when $k \neq 0$. The right hand side or inequality 3 is zero since the firm cannot win in the first auction, and hence, does not participate in the second auction.

Case 3: $\pi_{LH}^H - \epsilon = Max\{s_{H1}, s_{L1}\}$

In this case, the left hand side of the inequality is non-positive. The right hand side's expected payoff is $\kappa\epsilon > 0$ assuming that each firm wins the auction with probability κ.

Case 4: $\pi^H_{LH} + k\epsilon \geq \pi^H_{LH} - \epsilon > Max\{s_{H1}, s_{L1}\}$

In this case, $U((\pi^H_{LH} + k\epsilon, x), \sigma_H, \sigma_L) < U((\pi^H_{LH} - \epsilon, x), \sigma_H, \sigma_L)$. In the first auction, the H-firm we consider will win with a lower bid. This gives a higher payoff to H firm regardless of what happens in the second auction.

■

Proof of Proposition 2: We denote per period profit with π^i_j where $i = L, H$ shows the firm type and $j = LL, LH, LLH, LHH$ denotes how many and which firms are competing in the market. After straightforward calculations, equation 4, equation 5, and equation 6 show per period profit and the quantities produced by each type of firm when there are two firms and three firms in the market, respectively.

$$q^L_{LL} = \frac{A}{3b} \qquad \pi^L_{LL} = \frac{A^2}{9b} \qquad q^H_{LH} = \frac{A-2c}{3b} \qquad \pi^H_{LH} = \frac{(A-2c)^2}{9b} \tag{4}$$

$$q^L_{LLH} = \frac{A+c}{4b} \qquad \pi^L_{LLH} = \frac{(A+c)^2}{16b} \qquad q^H_{LLH} = \frac{A-3c}{4b} \qquad \pi^H_{LLH} = \frac{(A-3c)^2}{16b} \tag{5}$$

$$q^L_{LHH} = \frac{A+2c}{4b} \qquad \pi^L_{LHH} = \frac{(A+2c)^2}{16b} \qquad q^H_{LHH} = \frac{A-2c}{4b} \qquad \pi^H_{LHH} = \frac{(A-2c)^2}{16b} \tag{6}$$

Part A): We look for an equilibrium that does not involve weakly dominated strategies. In the first auction, Type H firms will bid until their profits are (almost) zero because of the competition between them. But their profits depend on whether L-firm will participate in the second auction or not (assuming that L-firm loses the first auction). Moreover, a strategy that involves bidding π^H_{LH} is weakly dominated by a strategy that involves bidding $\pi^H_{LH} - \epsilon$.[12] However, firm L can prevent the entry of H firms and maximize its payoff by bidding π^H_{LH} in the first auction.

[12]We note that we assume ϵ to be a sufficiently small monetary unit.

First let us calculate the bid of type H firms. Firm H knows that if its cost is low enough and it wins the first auction with a bid $\pi^H_{LH} - \epsilon = \frac{(A-2c)^2}{9b(1-\delta)} - \epsilon$, then firm L will not enter to the second auction. Because, if firm L enters the second auction, it has to pay the (smallest) reserve price and there will be three firms in the market and hence, its net profit will be negative when $0 < c < \frac{A}{11}$ as shown in equations below:

$$Net\ profit\ of\ Firm\ L = \pi^L_{LLH} - \pi^H_{LH} = \frac{(A+c)^2}{16b(1-\delta)} - \frac{(A-2c)^2}{9b(1-\delta)} < 0 \tag{7}$$

$$\Longleftrightarrow \frac{(A+c)^2}{16} < \frac{(A-2c)^2}{9} \tag{8}$$

$$\Longleftrightarrow \frac{(A+c)}{4} < \frac{(A-2c)}{3} \Longleftrightarrow 0 < c < \frac{A}{11} \tag{9}$$

Hence, type H firm will bid $\frac{(A-2c)^2}{9b(1-\delta)} - \epsilon$ in the first auction. Firm L will bid just $\frac{(A-2c)^2}{9b(1-\delta)}$ and win the first auction. Firm H will make a negative profit if it wins the second auction with a bid higher than the reservation price $\frac{(A-2c)^2}{9b(1-\delta)}$; hence, no firm H will participate in the second auction.

Part B): From Part A, we know that when $c \geq \frac{A}{11}$, firm L will enter the second auction. Hence, any firm H can bid at most $\pi^H_{LLH} - \epsilon = \frac{(A-3c)^2}{16b(1-\delta)} - \epsilon$. Firm L will bid $\frac{(A-3c)^2}{16b(1-\delta)}$ in the first auction. Because of the reservation price, no firm H will participate in the second auction.

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Proof of Proposition 4: When $\frac{A}{11} \leq c \leq \frac{A}{3}$, the bids in both auctions are the same but government will sell two licenses in the "no-reserve price" auction; hence, their revenue will be doubled. If $0 < c < \frac{A}{11}$, then in the original auction, government will sell one license but will make $\frac{(A-2c)^2}{9b(1-\delta)}$. In the no-reserve price, government will sell two licenses, each of them at a price of $\frac{(A-3c)^2}{16b(1-\delta)}$. When we compare the revenue, we get the following result.

$$\frac{(A-2c)^2}{9b(1-\delta)} > 2\frac{(A-3c)^2}{16b(1-\delta)} \Longleftrightarrow \frac{(A-2c)^2}{9} > 2\frac{(A-3c)^2}{16} \Longleftrightarrow \frac{(A-2c)}{3} > \sqrt{2}\frac{(A-3c)}{4} \tag{10}$$

$$\Longleftrightarrow 4A - 8c > 3\sqrt{2}A - 9\sqrt{2}c \Longleftrightarrow c > \frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} \tag{11}$$

■

Proof of Proposition 5: We consider social welfare in auctions as the sum of producer surplus, consumer surplus and government revenue from the auctions. We denote social welfare by W^i and government revenue by R^i, $i = 1, 2$. Where W^1 and R^1 denote social welfare and government revenue with the original auction, and W^2 and R^2 denote social welfare and government revenue with a no reserve price auction. According to Proposition 2 and 3, we know that $R^1 = \frac{(A-2c)^2}{9b(1-\delta)}$, when $0 < c < \frac{A}{11}$ or $R^1 = \frac{(A-3c)^2}{16b(1-\delta)}$, when $\frac{A}{11} \leq c \leq \frac{A}{3}$, and $R^2 = 2\frac{(A-3c)^2}{16b(1-\delta)}$. From Proposition 4, we know that the government will have more revenue by selling two licenses when $c < \frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} \approx \frac{A}{20}$ *or* $\frac{A}{11} \leq c \leq \frac{A}{3}$ and more revenue by selling one license when $\frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} < c < \frac{A}{11}$.

When $0 < c \leq \frac{A}{3}$, social welfare from the original auction will be,

$$W^1 = PS + CS + R^1 = 2 \cdot \frac{A^2}{9b} - R^1 + \frac{2A}{3b} \cdot \frac{1}{2} \cdot \frac{2A}{3} + R^1$$

$$W^1 = \frac{4A^2}{9b} \tag{12}$$

$$W^2 = PS + CS + R^2 = 2 \cdot \frac{(A+c)^2}{16b} + \frac{(A-3c)^2}{16b} - R^2 + \frac{3A-c}{4} \cdot \frac{1}{2} \cdot \frac{3A-c}{4b} + R^2$$

$$W^2 = \frac{15A^2 + 23c^2 - 10Ac}{32b} \tag{13}$$

Since $W^1 - W^2 = \frac{-7A^2-207c^2+90Ac}{288b} > 0$, when $(A-3c)(7A-69c) < 0$.

Thus, if $\frac{7A}{69} < c \leq \frac{A}{3}$, $W^1 > W^2$ and $R^2 > R^1$. If $0 < c < \frac{7A}{69}$, $W^2 > W^1$ and if $0 < c < \frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8}$ and $\frac{A}{11} \leq c \leq \frac{7A}{69}$, $R^2 > R^1$; if $\frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} < c < \frac{A}{11}$, $R^1 > R^2$.

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